## Homework \# 4

For problems 1-26, give a regular expression that accepts the specified language. Note that these are the same problems from homework \#3. Exercises to be handed in include 1,3 , and 10 .

1. The set of strings over $\{a, b, c\}$ in which all $a^{\prime} s$ preceded the $b^{\prime} s$, which in turn precede the $c^{\prime} s$.
2. The same as in 1 except for the null string.
3. The set of strings over $\{a, b\}$ in which the substring $a a$ occurs exactly once.
4. The set of strings over $\{a, b\}$ that do not contain the substring $a a a$.
5. The set of strings over $\{a, b, c\}$ that do not contain the substring $a a$.
6. The set of strings over $\{a, b, c\}$ that begin with an $a$, contain exactly two $b^{\prime} s$, and end with cc.
7. The set of strings over $\{a, b, c\}$ in which the total number of $b^{\prime} s$ and $c^{\prime} s$ is three.
8. The set of strings over $\{a, b, c\}$ in which every $b$ is followed by at least one $c$.
9. The set of strings over $\{a, b\}$ that contain the substring $a a$ and the substring $b b$.
10. The set of strings over $\{a, b, c\}$ that contain the substrings $a a, b b$, and $c c$.
11. The set of strings over $\{a, b, c\}$ with length three.
12. The set of strings over $\{a, b, c\}$ with length less than three.
13. The set of strings over $\{a, b, c\}$ with length greater than three.
14. The set of strings over $\{a, b\}$ in which the number of $a^{\prime} s$ is divisible by three.
15. The set of strings over $\{a, b\}$ in which every $a$ is either preceded or followed by a $b$, for example, $b a a b, a b a$, and $b$.
16. The set of strings over $\{a, b\}$ with an even number of $a^{\prime} s$ or an odd number of $b^{\prime} s$.
17. The set of strings over $\{a, b\}$ with an even number of $a^{\prime} s$ and an even number of $b^{\prime} s$.
18. The set of strings over $\{a, b\}$ that have odd length and contain exactly two $b^{\prime} s$.
19. The set of strings over $\{a, b, c\}$ that have odd length and contain exactly one $a$.
20. The set of strings over $\{a, b, c\}$ with an odd number of occurrences of the substring $a b$.
21. The set of strings over $\{a, b\}$ ending with the substring $a b b a$.
22. The set of strings over $\{1,2,3\}$ the sum of whose elements is divisible by 6 .
23. The set of strings over $\{a, b, c\}$ in which the number of a's plus the number of b's plus twice the number of $c$ 's is divisible by six.
24. The set of strings over $\{a, b\}$ in which every substring of length four has exactly one b .
25. The set of strings over $\{a, b\}$ that contain an even number of substrings $b a$.
26. Use the regular expression identities discussed in class to prove each of the following identities.
(a) $b a(b a)^{*}\left(a^{*} b^{*}+a^{*}\right)=(b a)^{*} b a a^{*}\left(b^{*}+\epsilon\right)$
(b) $b b^{*}\left(a^{*} b^{*}+\epsilon\right) b=b\left(b^{*} a^{*}+\epsilon\right) b b^{*}$
(c) $(a+b)^{*}=(a+b)^{*} b^{*}$
(d) $(a+b)^{*}=\left(a^{*}+b a^{*}\right)^{*}$
(e) $(a+b)^{*}=\left(b^{*}(a+\epsilon) b^{*}\right)^{*}$
27. The set of strings over $\{0,1\}$ in which every consecutive sequence of 3 symbols contains at least two zeros.
