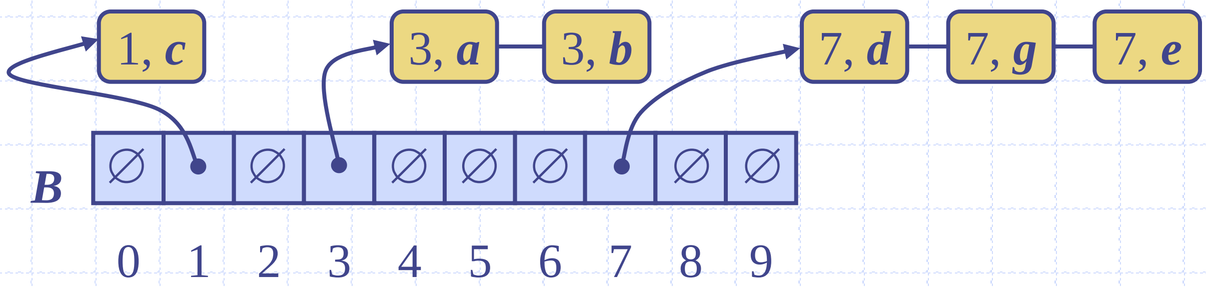
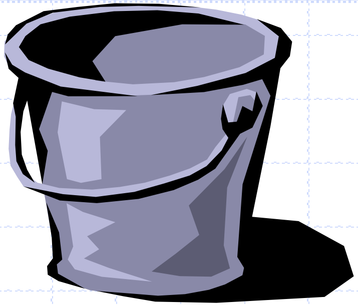
Presentation for use with the textbook Data Structures and Algorithms in Java, 6th edition, by M. T. Goodrich, R. Tamassia, and M. H. Goldwasser, Wiley, 2014

Bucket-Sort and Radix-Sort



Bucket-Sort

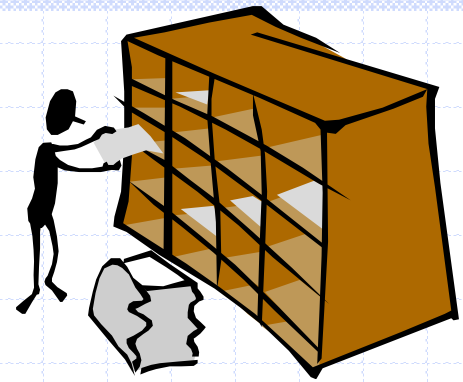


- Let be S be a sequence of n (key, element) items
  - with keys in the range $[0, N-1]$
- keys as indices into an auxiliary array B of sequences (buckets)

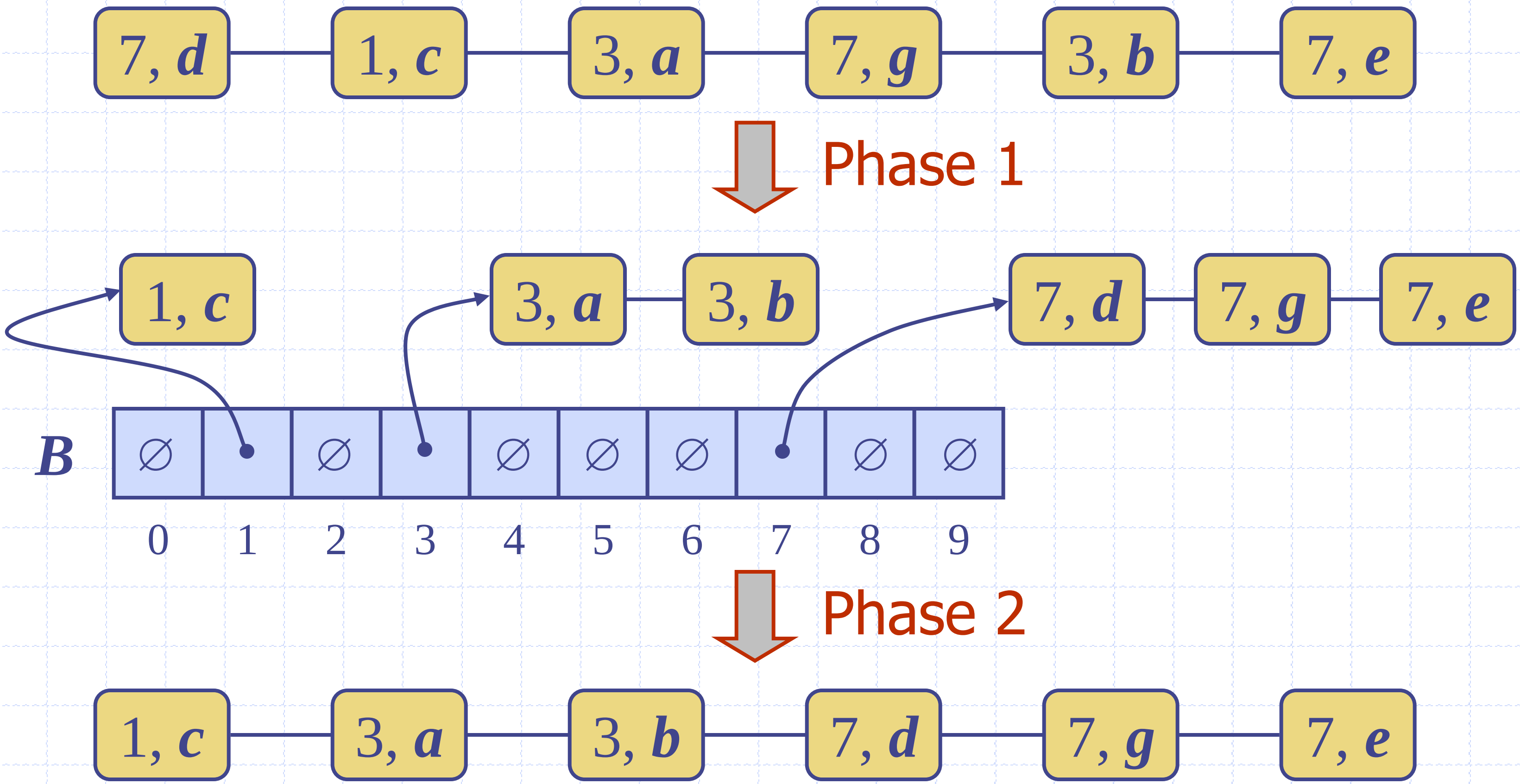
Phase 1: Empty sequence S by moving each entry (k, o) into its bucket $B[k]$

Phase 2: For $i = 0, \ldots, N-1$, move the entries of bucket $B[i]$ to the end of sequence S

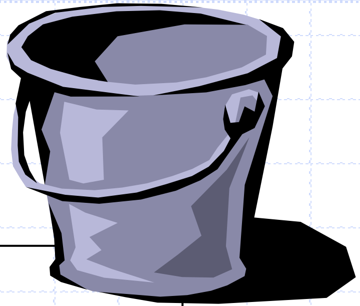
Example



- Key range [0, 9]



Bucket-Sort

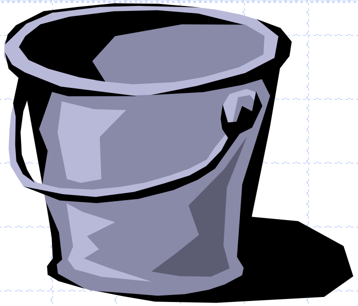


Algorithm bucketSort(S):
Input: Sequence S of entries with integer keys in the range [0, N – 1]
Output: Sequence S sorted in nondecreasing order of the keys

let B be an array of N sequences, each of which is initially empty

for each entry e in S **do** **// Phase 1**
  k = the key of e
  remove e from S
  insert e at the end of bucket B[k]
for i = 0 to N−1 **do** **// Phase 2**
  for each entry e in B[i] **do**
    remove e from B[i]
    insert e at the end of S

Performance Analysis



- n items, N buckets
- Time Complexity
  - Phase 1 takes $O(n)$ time
  - Phase 2 takes $O(n + N)$ time
- $O(n + N)$ time
- Linear time, faster than O(n log n) !
  - What is the catch?

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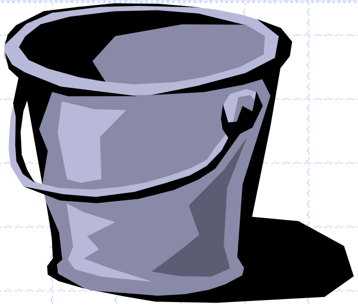

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- Linear time, faster than O(n log n) !
  - What is the catch?
  - O(n + N) space, not O(n) space
    - What if N buckets >> n items?

Properties

- Key-type Property
  - The keys are used as indices into an array and cannot be arbitrary objects
- Stable Sort Property
  - The relative order of any two items with the same key is preserved (before and after sorting)
  - Consider prices of a product and zip codes of the corresponding stores
    - Each zip code has multiple stores
    - Given a list of sorted prices
      - Sorting on zip codes doesn't affect the order of prices

Extensions

- Integer keys in the range $[a, b]$
  - Put entry (k, o) into bucket $B[k - a]$

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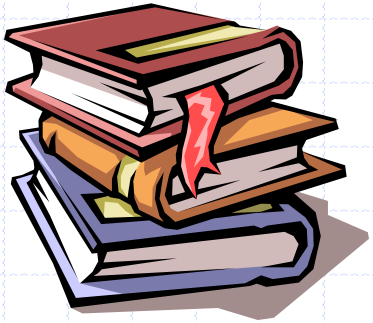
- String keys from a set D of possible strings, where D has constant size (e.g., names of the 50 U.S. states)
  - Sort D and compute the rank $r(k)$ of each string k of D in the sorted sequence
  - Put entry (k, o) into bucket $B[r(k)]$

Skipping the rest

Lexicographic Order



- A d-tuple is a sequence of d keys $(k_1, k_2, \ldots, k_d)$
  - key k_i is said to be the i-th dimension of the tuple
- Example:
  - The Cartesian coordinates of a point in space are a 3-tuple
- The lexicographic order of two d-tuples is recursively defined as follows

$$(x_1, x_2, \ldots, x_d) < (y_1, y_2, \ldots, y_d)$$
$$\Leftrightarrow$$
$$x_1 < y_1 \ \vee \ x_1 = y_1 \wedge (x_2, \ldots, x_d) < (y_2, \ldots, y_d)$$

I.e., the tuples are compared by the first dimension, then by the second dimension, etc.

Lexicographic-Sort

- C_i
  - comparator that compares two tuples by their i-th dimension
- $stableSort(S, C)$
  - a stable sorting algorithm that uses comparator C
- executing d times
  - *stableSort*
    - once per dimension
- $O(dT(n))$ time
  - $T(n)$ is the running time of *stableSort*

Algorithm *lexicographicSort*(S)
Input sequence S of d-tuples
Output sequence S sorted in lexicographic order

for $i \leftarrow d$ **downto** 1
  $stableSort(S, C_i)$

Example:

(7,4,6) (5,1,5) (2,4,6) (2, 1, 4) (3, 2, 4)

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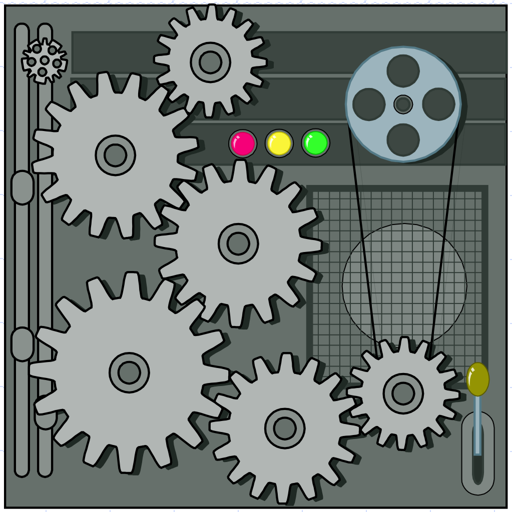
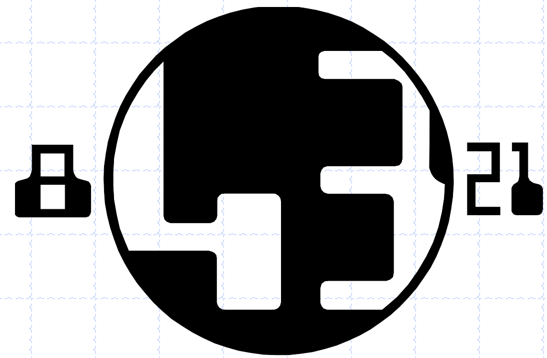
Radix-Sort



- specialization of lexicographic-sort
  - bucket-sort as the stable sorting algorithm
- keys in each dimension i are integers in the range $[0, N-1]$
- Radix-sort runs in time $O(d(n+N))$

Algorithm *radixSort*(S, N)
  Input sequence S of d-tuples such that $(0, \ldots, 0) \leq (x_1, \ldots, x_d)$ and $(x_1, \ldots, x_d) \leq (N-1, \ldots, N-1)$ for each tuple $(x_1, \ldots, x_d)$ in S
  Output sequence S sorted in lexicographic order
  for $i \leftarrow d$ **downto** 1
    bucketSort(S, N)

Radix-Sort for Binary Numbers



- n b-bit integers

  $$x = x_{b-1} \ldots x_1 x_0$$

- radix-sort with $N = 2$
- $O(bn)$ time
- For example, we can sort a sequence of 32-bit integers in linear time

Algorithm *binaryRadixSort*(S)
    Input sequence S of b-bit integers
    Output sequence S sorted
    replace each element x of S with the item $(0, x)$
    for $i \leftarrow 0$ **to** $b - 1$
        replace the key k of each item (k, x) of S with bit x_i of x
        bucketSort(S, 2)

Example

- Sorting a sequence of 4-bit integers

1001		0010		1001		1001		0001
0010		1110		1101		0001		0010
1101	→	1001	→	0001	→	0010	→	1001
0001		1101		0010		1101		1101
1110		0001		1110		1110		1110