History of Expression

We focus on the history of expression in linear, written media or language. Some of the important intellectual milestones are:

- Alphabet — page 3
- Algorithms and Number — page 15
- Notation and Logic — ??
THE EVOLUTION OF THE WRITTEN LANGUAGE

Cuneiform script  
Hieroglyphs  
The Latin Alphabet  
Emoticons

4000 BC  →  2000 BC  →  0  →  2000 ACD*

*(After Cognitive Disintegration)*
Development of the Alphabet

- **token** — a miniature of object, bullae
- **pictograph** — picture of object
  - **cuneiform** — wedge-shaped stylus, tomb of Darius the Great
  - **hieroglyphics** — hieratic (sacred), demotic (simplified), Rosetta stone (1799)
- **ideograph** — symbol for idea or object
- **logogram** — symbol for word or phrase
- **syllabary** — symbols for syllables of language
4.4 Hieroglyphic inscription of the thirteenth century

- throne: phonogram 'st'
- bread: phonogram 't'
- egg: determinative for female name
- goddess: determinative for female/goddess
  - Aset = Egyptian name of Isis
- swallow: phonogram 'wr' (rebus 'big')
- bread: phonogram 't', meaning: 'female'
- mouth: redundant phonogram 'r'
- weret = great woman
- vulture: phonogram 'mt' (rebus 'mother')
- cloth wrapped around pole: logogram 'ntr' ('God')
- bread: redundant phonogram 't'
- mut netcher = mother of God
- basket: logogram 'nb' ('master')
- bread: phonogram 't', meaning: 'female'
- sky: logogram 'pt' ('realm of gods')
- nebet pet = mistress of the sky

Aset weret mut netcher nebet pet = Isis, great woman, mother of God, mistress of the sky
Otl Aicher pictograms for the 1972 Munich Olympics. A pictogram, though simplified, depicts an object (or activity) in such a way that it can be identified without prior agreement, or common education.
In linguistics, the rebus principle is the use of existing symbols, such as pictograms, purely for their sounds regardless of their meaning, to represent new words. Many ancient writing systems used the rebus principle to represent abstract words, which otherwise would be hard to be represented by pictograms. An example that illustrates the Rebus principle is the representation of the sentence "I can see you" by using the pictographs of "eye–can–sea–ewe."
Acrophonic principle

[Greek ακρο- tip + φωνια voice, the initial sound]
In the history of writing this principle refers to the use of a written sign which originally took the form of a pictorial or logographic symbol of an object to represent the initial syllable or phoneme of the name of that object.
Strictly speaking, a pictogram represents by illustration, an ideogram represents an idea, and a logogram represents a word: Chinese characters are all logograms, but few are pictograms or ideograms. Casually, pictogram is used to represent all of these: it is a picture representing some concept.
A phonogram is a grapheme which represents a sound.
Evolution of logograms into phonogram.
Pictographic Origins of Cuneiform

The evolution of the cuneiform script

<table>
<thead>
<tr>
<th>Pictograms</th>
<th>Uruk c. 3100 BC</th>
<th>Ummetnasr c. 2800 BC</th>
<th>Classical Sumerian c. 2400 BC</th>
<th>Old-Akkadian c. 2200 BC</th>
<th>Old-Assyrian c. 1900 BC</th>
<th>Old-Babylonian c. 1700 BC</th>
<th>Neo-Assyrian c. 700 BC</th>
<th>Neo-Babylonian c. 600 BC</th>
<th>Picture</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neck + Head</td>
<td>Head front</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>Mouth + Nose + Tooth + Voice</td>
<td>Speak</td>
<td>Word</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shrouded Body (?)</td>
<td>Man</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>Sitting Bird</td>
<td>Bird</td>
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<tr>
<td>Bull's Head</td>
<td>Ox</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Star</td>
<td>Sky Heaven-God</td>
<td>God</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stream of Water</td>
<td>Water</td>
<td>Seed</td>
<td>Father Son</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orchard Greener to Grow</td>
<td>Orbe</td>
<td></td>
<td>Orchard Greener to Grow</td>
<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Land-Plot + Trees</td>
<td>Orbe</td>
<td></td>
<td>Orchard Greener to Grow</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Pictographic Origins of Cuneiform

The evolution of the cuneiform script

<table>
<thead>
<tr>
<th>Uruk</th>
<th>Sumerian</th>
<th>Akkadian</th>
<th>Assyrian</th>
<th>Babylonian</th>
<th>Assyrian</th>
<th>Babylonian</th>
<th>Picture</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>UPRIGHT</td>
<td>3100 BC</td>
<td>2400 BC</td>
<td>2200 BC</td>
<td>1900 BC</td>
<td>1700 BC</td>
<td>700 BC</td>
<td>600 BC</td>
<td>NECK + HEAD</td>
</tr>
<tr>
<td></td>
<td>TURNED 90° TO LEFT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>MOUTH NOSE TOOTH SPREAD WORD</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>SHROUDED BODY (?)</td>
<td>MAN</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SITTING BIRD</td>
<td>BIRD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>BULL'S HEAD</td>
<td>OX</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>STAR</td>
<td>SKY HEAVEN GOD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>STREAM OR WATER</td>
<td>WATER SEED FATHER SON</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LAND + PLOT</td>
<td>ORCHARD GREENERY TO GROW TO WRITE</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>TREES</td>
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</tbody>
</table>
Pictographic Origins of Chinese

<table>
<thead>
<tr>
<th></th>
<th>oracle bone</th>
<th>greater seal</th>
<th>lesser seal</th>
<th>clerkly script</th>
<th>standard script</th>
<th>running script</th>
<th>cursive script</th>
<th>modern simplified script</th>
</tr>
</thead>
<tbody>
<tr>
<td>rén (*nin)</td>
<td>人</td>
<td>人</td>
<td>人</td>
<td>人</td>
<td>人</td>
<td>人</td>
<td>人</td>
<td>人</td>
</tr>
<tr>
<td>nǚ (*nraʔ)</td>
<td>女</td>
<td>女</td>
<td>女</td>
<td>女</td>
<td>女</td>
<td>女</td>
<td>女</td>
<td>女</td>
</tr>
<tr>
<td>ér (*nhaʔ)</td>
<td>耳</td>
<td>耳</td>
<td>耳</td>
<td>耳</td>
<td>耳</td>
<td>耳</td>
<td>耳</td>
<td>耳</td>
</tr>
<tr>
<td>mǎ (*mrāʔ)</td>
<td>马</td>
<td>马</td>
<td>马</td>
<td>马</td>
<td>马</td>
<td>马</td>
<td>马</td>
<td>马</td>
</tr>
<tr>
<td>yú (*ŋha)</td>
<td>鱼</td>
<td>鱼</td>
<td>鱼</td>
<td>鱼</td>
<td>鱼</td>
<td>鱼</td>
<td>鱼</td>
<td>鱼</td>
</tr>
<tr>
<td>shān (*srān)</td>
<td>山</td>
<td>山</td>
<td>山</td>
<td>山</td>
<td>山</td>
<td>山</td>
<td>山</td>
<td>山</td>
</tr>
<tr>
<td>rì (*nit)</td>
<td>日</td>
<td>日</td>
<td>日</td>
<td>日</td>
<td>日</td>
<td>日</td>
<td>日</td>
<td>日</td>
</tr>
<tr>
<td>yuè (*ŋot)</td>
<td>月</td>
<td>月</td>
<td>月</td>
<td>月</td>
<td>月</td>
<td>月</td>
<td>月</td>
<td>月</td>
</tr>
<tr>
<td>yǔ (*waʔ)</td>
<td>雨</td>
<td>雨</td>
<td>雨</td>
<td>雨</td>
<td>雨</td>
<td>雨</td>
<td>雨</td>
<td>雨</td>
</tr>
<tr>
<td>yún (*wən)</td>
<td>雲</td>
<td>雲</td>
<td>雲</td>
<td>雲</td>
<td>雲</td>
<td>雲</td>
<td>雲</td>
<td>雲</td>
</tr>
</tbody>
</table>
# Pictographic Origins of Greek

<table>
<thead>
<tr>
<th>Old Phoenician</th>
<th>Classical Greek</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Letter</strong></td>
<td><strong>Name</strong></td>
</tr>
<tr>
<td>aleph</td>
<td>ox</td>
</tr>
<tr>
<td>beth</td>
<td>house</td>
</tr>
<tr>
<td>gimel</td>
<td>camel</td>
</tr>
<tr>
<td>daleth</td>
<td>door</td>
</tr>
<tr>
<td>heth</td>
<td>E</td>
</tr>
<tr>
<td>zain</td>
<td>weapon</td>
</tr>
<tr>
<td>theth</td>
<td>arm</td>
</tr>
<tr>
<td>kaph</td>
<td>palm of hand</td>
</tr>
<tr>
<td>lamed</td>
<td>goad</td>
</tr>
<tr>
<td>mem</td>
<td>water</td>
</tr>
<tr>
<td>nun</td>
<td>fish</td>
</tr>
<tr>
<td>sāmekh</td>
<td>fish</td>
</tr>
<tr>
<td>shin</td>
<td>tooth</td>
</tr>
<tr>
<td>taw</td>
<td>cross mark</td>
</tr>
<tr>
<td>waw</td>
<td></td>
</tr>
</tbody>
</table>

*9.1 The Old Phoenician models for some of the letters of the Classical Greek alphabet*
Development of the Alphabet

- consonantal alphabet emerged on the acrophonic principle in Semitic languages
- Greek (an Indo-European language) replaced unused consonants with vowels

### Evolution of Greek and Latin Alphabets

<table>
<thead>
<tr>
<th>Phoenician</th>
<th>א ב ג ד ח ו י ז ק ל מ נ א צ ד פ ו י צ פ א כ ו מ נ א צ ד פ ו י צ פ א כ ו מ נ א צ ד פ ו י צ פ א כ ו מ נ א צ ד פ ו י צ פ א כ ו מ נ א צ ד פ ו י צ פ א כ ו מ נ א צ ד פ ו י צ פ א כ ו מ נ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early Greek</td>
<td>A B ג D E F Z H Θ Ι Κ Λ Μ Ν Ξ Ο Π Φ Χ Ψ Ω</td>
</tr>
<tr>
<td>Later Greek</td>
<td>A B Γ D E F Z H Θ Ι Κ Λ Μ Ν Ξ Ο Π Ψ Ω</td>
</tr>
<tr>
<td>Etruscan</td>
<td>Θ Γ Д Е F Z H Θ Ι К Λ М Н Ξ О Π Θ Π Ψ Ω</td>
</tr>
<tr>
<td>Early Latin</td>
<td>A B C G D E F Z H I K L M N O P Q R S T V Y X</td>
</tr>
<tr>
<td>letter</td>
<td>name</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>$A\alpha$</td>
<td>alpha</td>
</tr>
<tr>
<td>$B\beta$</td>
<td>beta</td>
</tr>
<tr>
<td>$\Gamma\gamma$</td>
<td>gamma</td>
</tr>
<tr>
<td>$\Delta\delta$</td>
<td>delta</td>
</tr>
<tr>
<td>$E\epsilon$</td>
<td>epsilon</td>
</tr>
<tr>
<td>$Z\zeta$</td>
<td>zeta</td>
</tr>
<tr>
<td>$H\eta$</td>
<td>eta</td>
</tr>
<tr>
<td>$\Theta\theta$</td>
<td>theta</td>
</tr>
<tr>
<td>$I\iota$</td>
<td>iota</td>
</tr>
<tr>
<td>$K\kappa$</td>
<td>kappa</td>
</tr>
<tr>
<td>$\Lambda\lambda$</td>
<td>lambda</td>
</tr>
<tr>
<td>$M\mu$</td>
<td>mu</td>
</tr>
</tbody>
</table>
Development of Mathematical Language

- Babylonians: algorithm, base 60, positional notation
- Greeks: Euclidean algorithm, proof and logic
- Arabs: Abu Jáfar Mohammed ibn Musa al-Khorezmi (780–850), algorithm, algebra
- The zero and the decimal system. Fibonacci, et al.
Early Babylonian Algorithm

Length and width is to be equal to the area. You should proceed as follows. Make two copies of one parameter. Subtract 1. Form the reciprocal. Multiply by the parameter you copied. This gives the width.

In modern terms: if $x + y = xy$, then compute $y$ by $y = (x - 1)^{-1}x$.

Source: Otto Neugebauer, *The Exact Sciences in Antiquity*. 
The Babylonians of the Amorite period invented place-value or positional notation.

The prodigious power of number as a language was revealed in this momentous grammatical breakthrough: position in the expression governs value. American mathematician Oswald Veblen was to call it a singular achievement of the human intellect, and Otto Neugebauer, “undoubtedly one of the more fertile inventions of humanity,” comparable to the “the invention of the alphabet as contrasted to the use of thousands of picture signs.”

Everyone who has been exposed to elementary geometry will doubtless recall that it is taught as a deductive discipline. It is not presented as an experimental science whose theorems are to be accepted because they are in agreement with observation. This notion, that a proposition may be established as the conclusion of an explicit logical proof, goes back to the ancient Greeks, who discovered that is known as the “axiomatic method” and used it to develop geometry in a systematic fashion.

Euclid’s Algorithm

**Proposition.** Given two positive integers, find their greatest common divisor.
Let $A$, $C$, be the two given positive integers; it is required to find their greatest common divisor. If $C$ divides $A$, then $C$ is a common divisor of $C$ and $A$, since it also divides itself. And it clearly is in fact the greatest, since no greater number than $C$ will divide $C$.
But if $C$ does not divide $A$, then continually subtract the lesser of the numbers $A$, $C$ from the greater, until some number is left that divides the previous one. This will eventually happen, for if unity is left, it will divide the previous number.
Now let $E$ be the positive remainder of $A$ divided by $C$; let $F$ be the positive reminder of $C$ divided by $E$
Leonardo da Pisano was the son of Guilielmo and a member of the Bonacci family. He is best known by a name he never used—Fibonacci. His father was head of one of Pisa’s overseas custom houses in Bugia on the coast of North Africa (now Bejaïa, Algeria) and he purposely had his son taught decimal arithmetic. Fibonacci wrote Liber abaci (Book of the Abacus) first published in 1202 which opens with the sentence:

These are the nine figures of the Indians 9 8 7 6 5 4 3 2 1. With these nine figures, and with this sign 0 which in Arabic is called zephirum, any number can be written, as will below be demonstrated.
Zero

Notice the order of the digits: right to left, as in Arabic. It is as if Fibonacci feared that writing them left to right, as in Latin, would somehow diminish their magic. An air of mystery remained for centuries with those that knew algorism, the art of reckoning with decimal digits. This is responsible for the word “cipher,” meaning to code a message into symbols to hide its meaning. Although less common today, the phrase “to cipher” means to do arithmetic. Actually Fibonacci’s book itself was not particularly influential. Other books, primarily Carmen de Algorismo (The Poem of Algorism) by Alexander de Villa Dei, were apparently more influential. The Carmen de Algorismo dates back to about the year 1220 and was entirely in hexameter verse. (A verse of poetry consisting of six rhythmic units known as dactyl in a row. A dactyl has one long syllable plus two short syllables.)
Pre-19th Century

- Johann Widman, 1489, “+” and “-”
- Robert Recorde, 1557, “=”
- William Oughtred, 1631, “×”
- James Hume, 1636, $x^2$
- Johann Rahn, 1659, ÷
- William Jones, 1706, $\pi$
Pre-19th Century

Gottfried Wilhelm Leibniz (1646–1716) search for a method he called \textit{characteristica generalis} or \textit{lingua generalis}.

\begin{quote}
I would like to give a method \ldots in which all truths of the reason would be reduced to a kind of calculus. This could at the same time be a kind of language or universal script, but very different from all that have been projected hitherto, because the characters and even the words would guide reason, and the errors (except those of fact) would only be errors of computation. It would be very difficult to form or invent this Language or Characteristic, but very easy to learn it without any Dictionaries.
\end{quote}
Gottlob Frege (1848–1925)

Frege was one of history’s most important mathematicians and contributed to making mathematics precise and formal.

- Truth-functional propositional calculus
- Theory of quantification
- Derivation according to form

Ironically his formalistic achievements have a fundamental flaw which make the work all the more interesting and significant.

Frege studied and later taught at the University of Jena. Frege invented a two-dimensional notation for logic that was difficult to typeset.
Gottlob Frege (1848–1925)
Quantifiers

The ambiguity of statements such as “Every philosopher admires some logician” is difficult to express in syllogistic theory, but in Fregean logic is readily reflected in the differing scope of the quantifiers. In modern notation, the two readings would be formalized as follows:

1. \((\forall x)(P x \Rightarrow (\exists y)(L y \& A xy))\). [For all \(x\), if \(x\) is a philosopher, then there is some \(y\) such that \(y\) is a logician and \(x\) admires \(y\).]

2. \((\exists y)(L y \& (\forall x)(P x \Rightarrow A xy))\). [There is some \(y\) such that \(y\) is a logician and for all \(x\), if \(x\) is a philosopher, then \(x\) admires \(y\).]

The ambiguity of statements such as “Every philosopher admires some logician” is difficult to express in syllogistic theory, but in Fregean logic is readily reflected in the differing scope of the quantifiers. In modern notation, the two readings would be formalized as follows:

1. For all $x$, if $x$ is a philosopher, then there is some $y$ such that $y$ is a logician and $x$ admires $y$.

$$\forall x (Px \Rightarrow \exists y (Ly \& Axy))$$

2. There is some $y$ such that $y$ is a logician and for all $x$, if $x$ is a philosopher, then $x$ admires $y$.

$$\exists y (Ly \& \forall x (Px \Rightarrow Axy))$$

Grundgesetze der Arithmetik, volume 2, 1903, page 178. “The comfort of the typesetter is certainly not the *summum bonum*” — G. Frege, 1897
<table>
<thead>
<tr>
<th>Basic concept</th>
<th>Frege's notation</th>
<th>Modern notations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judging</td>
<td>$\vdash A$, $\Vdash A$</td>
<td>$p(A) = 1$ $p(A) = i$</td>
</tr>
<tr>
<td>Negation</td>
<td>$\neg A$</td>
<td>$\neg A$ $\neg A$</td>
</tr>
<tr>
<td>Conditional</td>
<td></td>
<td>$B \rightarrow A$</td>
</tr>
<tr>
<td>(implication)</td>
<td>$\underline{A}$</td>
<td>$B \supset A$</td>
</tr>
<tr>
<td>Universal quantification</td>
<td></td>
<td>$\forall y: \Phi(y)$</td>
</tr>
<tr>
<td>Existential quantification</td>
<td></td>
<td>$\exists y: \Phi(y)$</td>
</tr>
<tr>
<td>Content identity (equal sign)</td>
<td>$A \equiv B$</td>
<td>$A = B$</td>
</tr>
</tbody>
</table>
Es kommt hier darauf an, sich klar zu machen, was Definiren ist und was dadurch erreicht werden kann. Man scheint ihm vielfach eine schöpferische Kraft zuzutrauen, während doch dabei weiter nichts geschieht, als dass etwas abgrenzend hervorgehoben und mit einem Namen bezeichnet wird. Wie der Geograph kein Meer schafft, wenn er Grenzlinien zieht und sagt: den von diesen Linien begrenzten Theil der Wasserfläche will ich Gelbes Meer nennen, so kann auch der Mathematiker durch sein Definiren nichts eigentlich schaffen. Man kann auch nicht einem Dinge durch blosse Definition eine Eigenschaft anzaubern, die es nun einmal nicht hat, es sei denn die eine, nun so zu heissen, wie man es etwa benannt hat.

Frege, Gesetze Arithmetik, page xiii
The geographer does not create a sea when he draws border lines and says: The part of the surface of the ocean delimited by these lines, I am going to call the Yellow Sea; and, neither can the mathematician really create anything by his act of definition. Nor can one by pure definition magically conjure into a thing a property that in fact it does not possess—save that of now being called by the name with which one has named it.
There is some irony for me in the fact that the man about whose philosophical views I have devoted, over years, a great deal of time to thinking, was, at least at the end of his life, a virulent racist, specifically an anti-semite. This fact is revealed by a fragment of a diary which survives among Frege’s Nachlass, but which was not published with the rest by Professor Hans Hermes in Freges nachgelassene Schriften. The diary shows Frege to have been a man of extreme right-wing political opinions, bitterly opposed to the parliamentary system, democrats, liberals, Catholics, the French and, above all, Jews, who he thought ought to be deprived of political rights and, preferably, expelled from Germany. When I first read that diary, many years ago, I was deeply shocked, because I had revered Frege as an absolutely rational man, if, perhaps, not a very likable one. I regret that the editors of Frege’s Nachlass chose to suppress that particular item. From it I learned something about human beings which I should be sorry not to know; perhaps something about Europe, also.

Betrand Russell (1872–1970)

Russell and Whitehead took over 300 pages to prove $1 + 1 = 2$. Volume II of the first edition, page 86. Underneath is the comment, “The above proposition is occasionally useful.”
Another page from PM.
Russell’s Paradox

Russell found a flaw in Frege’s formalization of mathematics.

In a remote village all the men shave themselves or they go to the village barber who shaves them. One day the municipal authorities issued the following directive: the village barber is required to shave all (and only) the men who do not shave themselves.

The barber, if male, cannot comply. If he doesn’t shave himself, he, as the barber, is ordered to shave himself. If he does shave himself, then he is ordered not to.

Mistakes are an essential part of education. 

Ertrand Russell, British philosopher

从错误中吸取教训是教育极为重要的

英国哲学家 罗素 B.
Réné Magritte
This is not a pipe
DID YOU MAKE A NEW YEAR'S RESOLUTION, ERNIE?

I RESOLVED NOT TO MAKE ANY NEW YEAR'S RESOLUTIONS.

RUSSELL'S PARADOX

RUSSELL WHO?

HELLO FOLKS - FOR THOSE WHO DIDN'T UNDERSTAND YESTERDAY'S REFERENCE TO RUSSELL'S PARADOX, I'LL EXPLAIN WHAT IT IS...

BERTRAND RUSSELL (1872-1970) WAS A BRITISH MATHEMATICIAN/PHILOSOPHER/WRITER...

RUSSELL'S PARADOX

AS AN AID TO MY FOREIGN TRANSLATORS, "PARADOX" SOUNDS LIKE "PAIR OF DOCS", I.E. "DOCTORS"! GET IT?!

GET IT?!

IT'S VERY FUNNY IN ENGLISH.
I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that, if certainty were indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure. . . .
... But as the work proceeded, I was continually reminded of the fable about the elephant and the tortoise. Having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.

Bertrand Russell
“Reflections on my Eightieth Birthday”
Portraits from Memory, 1956, page 54
After a lecture on cosmology and the structure of the solar system, [William] James was accosted by a little old lady.

“Your theory that the sun is the centre of the solar system, and the earth is a ball which rotates around it has a very convincing ring to it, Mr. James, but it’s wrong. I’ve got a better theory,” said the little old lady.

“And what is that, madam?” inquired James politely.

“That we live on a crust of earth which is on the back of a giant turtle.”

Not wishing to demolish this absurd little theory by bringing to bear the masses of scientific evidence he had at his command, James decided to gently dissuade his opponent by making her see some of the inadequacies of her position.

“If your theory is correct, madam,” he asked, “what does this turtle stand on?”

“You’re a very clever man, Mr. James, and that’s a very good question,” replied the little old lady, “but I have an answer to it. And it’s this: The first turtle stands on the back of a second, far larger, turtle, who stands directly under him.”

“But what does this second turtle stand on?” persisted James patiently.

To this, the little old lady crowed triumphantly,

“It’s no use, Mr. James it’s turtles all the way down.”

John R. Ross, *Constraints on Variables in Syntax*, 1967
“It’s turtles all the way down.”
Friedrich Dürrenmatt (1921–1990)
Swiss playwright

19. Reality appears in the paradoxes. *The Physiker*

19. Im Paradoxen erscheint die Wirklichkeit. *The Physichs*
Constructive Mathematics

The period of acrimonious relations between the mathematicians David Hilbert (1862–1943) and L. E. J. Brouwer (1881–1966) is now known as the War of the Frogs and the Mice after Albert Einstein’s 1920s characterization of it as a Frosch-Musekrieg (frog and mice battle).

John Hays wrote a fable about Hilbert (the mouse) and Brouwer (the frog) “The Battle of the Frog and the Mouse”, which appeared in 1984 in the Mathematical Intelligencer (reprinted in 1992 in Pi in The Sky by John D. Barrow). A silly altercation is called a batrachomyomachia (Greek βατραχόμομαξια, frog, μυς, mouse, and, μαχαξ, battle), the Battle of Frogs and Mice, after an ancient comic parody on the Iliad of that name.
Alfred Tarski (1902–1983)

\[ M[A \land B] = M[A] \text{ and } M[B] \]
Alfred Tarski (1902–1983)

Tarski made important contributions in many areas of mathematics, including metamathematics, set theory, measure theory, model theory, and general algebra.

Find out more out at MacTutor History of Mathematics
That these signs may be convenient is very possible, but that they should be destined to change the face of the whole philosophy is quite another matter. It is difficult to admit that the word if, when written $\supset$, a virtue it did not possess when written if.

Henri Poincare
Programming Formalisms

- Turing machines
- Lambda calculus
- Post system
- Herbrand-Gödel equations
- Universal register machine
- and others ...
A programming language is Turing complete if all computational algorithms that are able to be performed, can be performed in that language. All the programming languages we discuss are Turing complete.

Does that mean that there is no point in discussing programming languages?
Turing Complete

Theoretically all languages are the same (are Turing complete). Just like all buildings are the same (they keep the rain out), and all bridges are the same (they connect a place with another).
Alan Mathison Turing (1912–1954)

Find out more out at MacTutor History of Mathematics

Andrew Hodges, *Alan Hodges: The Enigma.*
Alan Mathison Turing (1912–1954)

As Turing wrote in *Undecidable*, p. 128 (italics added):

> It is possible to invent a single machine which can be used to compute any computable sequence. If this machine $U$ is supplied with the tape on the beginning of which is written the string of quintuples separated by semicolons of some computing machine $M$, then $U$ will compute the same sequence as $M$.

This finding is now taken for granted, but at the time (1936) it was considered astonishing. The model of computation that Turing called his “universal machine”—”U” for short—is considered by some (cf Davis (2000)) to have been the fundamental theoretical breakthrough that led to the notion of the stored program computer. In the words of Minsky (1967), p. 104:

> Turing’s paper ... contains, in essence, the invention of the modern computer and some of the programming techniques that accompanied it.
A Turing machine is a 7-tuple \( \langle Q, T, I, \delta, b, q_0, q_f \rangle \) where

1. \( Q \) is the set of states,
2. \( T \) is the set of tape symbols,
3. \( I \) is the set of input symbols, \( I \subseteq T \),
4. \( \delta : Q \times T \rightarrow Q \times T \times \{L, R\} \) is the transition function,
5. \( b \in T \setminus I \) is the designated symbol for a blank (the symbol always beyond the right edge of the one-way infinite tape),
6. \( q_0 \in Q \) is the initial state, and
7. \( q_f \in Q \) is the final or accepting state.
Turing’s original paper contains a programming language, just as Gödel’s paper does, or what we would now call a programming language, but these two programming languages are very different. Turing’s isn’t a high-level language like LISP; it’s more like a machine language, the raw code of ones and zeros that are fed to a computer’s central processor. Turing’s invention of 1936 is, in fact, a horrible machine language, one that nobody would want to use today, because it’s too rudimentary.

Alonzo Church (1903–1995)

Alonzo Church was born on 14 June 1903 in Washington, D.C. and died 11 August 1995 in Hudson, Ohio. Church was professor of mathematics at Princeton University from 1929 to 1967 when he became professor of mathematics and philosophy at UCLA.

- Church’s Theorem (1936), showing that arithmetic is undecidable.
- Church’s Thesis, conjecturing that effective computation is equivalent to the notion of a “recursive” function.
- The Lambda Calculus.

Find out more out at MacTutor History of Mathematics
What is a programming language?
What is a programming language?

General purpose! Not Visual Basic (drag-and-drop), XML (mark-up), RPG (Report Program Generator), APT (Automatically Programmed Tools), Macsyma, . . .