Rev. Doctor Syntax
Doctor Syntax was the fictitious hero of several 19th century satirical poems by William Combe (1742–1823). The publications were illustrated by Thomas Rowlandson (1756–1827).

Rowlandson’s many comic illustrations offered humorous commentary on the political and social conventions of his day. Rowlandson is perhaps best known for his animated drawings of the mis-adventures of the infamous Dr. Syntax. The various escapades of the fictional English clergyman and schoolmaster, Dr. Syntax, are chronicled in three volumes or “Tours." The Tours of Dr. Syntax were satires on a then contemporary series of picturesque adventures to various parts of England with an emphasis on landscape illustration and poetic verse.

- The tour of Doctor Syntax in search of the picturesque
- The second tour of Doctor Syntax in search of consolation
- The third tour of Doctor Syntax in search of a wife
Overview of Syntax

- Study of Language — page 4
- Structure — page 10
- Formal Languages — page 30
- Regular Expressions — page 62
- BNF, * Context Free Grammar, parse trees — page 106
- *Attribute Grammars, *Post Systems
Semiotics. Date: 1880. A general philosophical theory of signs and symbols that deals especially with their function in both artificially constructed and natural languages. The application of linguistic methods to other than natural languages. A philosophy that emphasizes the significance of language as universal.

In his 1946 book, *Signs, Language, and Behavior*, American philosopher Charles William Morris (1901–1979) gave these definitions for the branches of the science of signs:

- **pragmatics** — origin, uses and effects of signs with the behavior in which they occur;
- **semantics** — the signification of signs in all modes of signifying;
- **syntax** — combination of signs without regard for their specific significations or their relations to the behavior in which they occur.
Study of Language

- **syntax** is the form, structure of language
- **semantics** is the meaning of language
- **pragmatics** is origin, use, context of language
We are quick to dismiss pragmatics which lacks much in the way of unifying principles, but the true significance of a language and the productivity of the programming users rests on history, context, and *milleu* of the language—the “ecosystem” of tools, documentation, society of users, etc.

Distinction between semantics and syntax:

1. number versus numeral
2. character versus glyph
3. conjunction versus $\land$
'Twas brillig, and the slithy toves
    Did gyre and gimble in the wabe;
All mimsy were the borogoves,
    And the mome raths outgrabe.

Lewis Carroll, “Jabberwocky”

We know certain things—for instance, that all this happened in the past and that there was more than one tove but only one wabe. We know these things because of the grammatical structure of the English language.
Can you translate the poem “Jabberwocky” into other languages?
Can you translate the poem “Jabberwocky” into other languages?

’Twas BRKPT and the I/O queue
   Was SYMMING FASTRAND like the wind.
All idle was the CAU
   As the last run had just FINNED.

“Beware the UNIVAC, my son,
   Its FASTRAND and its high-speed drum,
And FIELDATA, and listen for
   The CTMC’s hum.”

Harry Gilbert, mid 1970s
Twas brillig, and the Protocols
    Did USER-SERVER in the wabe.
All mimsey was the FTP,
    And the RJE outgrabe,

Beware the ARPANET, my son;
    The bits that byte, the heads that scratch;
Beware the NCP, and shun
    the frumious system patch,
Structure

The arrangement of elements of an entity and their relationships to each other.

- Linear structure – like links of a chain
- Hierarchical structure – like nested mixing bowls, or boxes within boxes
- Trees – like family tree, or railroad yard

Grammars are a way of describing these and more complex structures.
Linear Structure

Links in a chain
Hierarchical Structure

Nested mixing bowls
Tree Structure

Railroad yard
Even though it is invisible, structure has great importance, even meaning. Words, of course, have meaning. And sometimes they have two meanings leading to ambiguity.

The captain corrected this list.

What list? The listing of the ship or some ship’s document? This is an example of *lexical ambiguity*.

Even if none of the words are ambiguous, a sentence may be ambiguous if the invisible structure relating the words is ambiguous. This is called *structural ambiguity*.
Sentence structure is crucial to disambiguate the following sentences.

1. The chicken is ready to eat.
Sentence structure is crucial to disambiguate the following sentences.

1. The chicken is ready to eat.
   Who is eating?
2. He saw that gasoline can explode.

Who is eating?
Sentence structure is crucial to disambiguate the following sentences.

1. The chicken is ready to eat.
   Who is eating?

2. He saw that gasoline can explode.
   What is exploding?

3. Flying planes can be dangerous.
Sentence structure is crucial to disambiguate the following sentences.

1. The chicken is ready to eat.
   Who is eating?

2. He saw that gasoline can explode.
   What is exploding?

3. Flying planes can be dangerous.
   What is dangerous?
CS Theory Classes

- Complexity
- Computability
- Efficiency
- Formal Languages, Automata
- Models
- Limits
The science of syntax is about unambiguous communication and as such is neutral. Let us at least be clear. Hard to define “good” and “bad” syntax. Only experience helps, and that takes time. The lessons of programming language design are ad hoc.
DO 10 I = 1.5
   A(I) = X + B(I)
10  CONTINUE

IF IF=THEN THEN THEN=ELSE; ELSE ELSE=IF
IF IF THEN THEN=ELSE; ELSE ELSE=0;

while ( ... ) {
   ...
   break;
   ...
}
In the study of compilers there are many fine-grained distinctions made about grammars. These distinction might point out features that humans might well avoid. The following is not LL, but is LR:

```
stm ::= "if" expr "then" stm "else" stm
     | "if" expr "then" stm
```
Lexical Structure Versus Phrase Structure

In lexical structure the alphabet used is a set of characters (e.g., Latin1) and the goal is to categorize substring into different tokens each described by their own language.

In phrase structure, the alphabet used is a set tokens, and one goal is to determine if the program is syntactically correct (i.e., is the phrase in the language or not). Actually the goal is more broad than that. The goal is to identify the phrase structure or derivation of syntactically correct programs.
with Ada.Text_IO;
use Ada;

-- program to do one thing: print a message
procedure World is
  -- Some text given a name for later use
  Msg: constant String := "Hello world!";
begin
  Text_IO.Put_Line (Msg);
end World;
Program as Stream of Characters

```
with Ada.Text_IO;
use Ada;

-- program to do one thing: print a message
procedure World is
  -- Some text given a name for later use
  Msg: constant String := "Hello world!";
begin
  Text_IO.Put_Line (Msg);
end World;
```
Program as Stream of Tokens

```
with Ada.Text_IO;
use Ada;

-- program to do one thing: print a message
procedure World is
  -- Some text given a name for later use
  Msg: constant String := "Hello world!";
begin
  Text_IO.Put_Line (Msg);
end World;
```

```
with --begin 2 * x;
```
Program as Tree Structure

```
with Ada.Text_IO;
use Ada;

-- program to do one thing: print a message
procedure World is
    -- Some text given a name for later use
    Msg: constant String := "Hello world!";
begin
    Text_IO.Put_Line (Msg);
end World;
```

with --begin 2 * x;
Free-format as opposed to one statement per line. In earlier versions of FORTRAN a fixed lexical structure of one statement per punch card stating on column 7. COBOL and Basic also used to be fixed format.

For the most part languages today permit the programmer to choose the white space (and hence the layout) of the program. Some layouts are clearly more legible and others, so the need for style guidelines. Python, Haskell, and Occam require proper indenting.

Should a language specify the layout (use of white space)?
“Hello World!” in Whitespace
“Hello World!” in Whitespace (S=space, T=tab, L=line feed)
Natural language is full of unresolvable differences and “jagged edges.” (This makes it rich and interesting.)
Formal Language Versus Natural Language
First some definitions:

- *object language*, the language being studied and *meta language*, the language used to communicate
- *symbol*
- *alphabet* is a set of symbols
- *strings* is an ordered sequence of zero, one or more symbols
- *formal language* is a set of strings

Unlike natural languages, it is unambiguous for a formal language $L$ if $s \in L$ or $s \notin L$. The key challenge is to find good (finite) representation of (infinite) sets.
Object and Meta Language

*Object language versus meta-language.* The *object language* is the language being studied, and the *meta-language* is the language being used to communicate. For example, in the class French 101 one may study the French language, it is the object language. To communicate to the students it may be necessary for the teacher to speak in English. English is the meta-language in this case.
To be an Atom is to be an atom.

The Little Typer, 2018, page 3
What is a *symbol*?

In the study of formal languages, for example, we define an *alphabet* as a finite set of symbols, for example, \{0, 1\} or \{a, b, c\}.

To make the definition understandable we focus on the meaning of the word “symbol” (we assume a finite set is understandable).

These primitive, or *a priori* in the sense of Kant, concepts like symbol are sometimes awkward to define.

We begin with a simple definition and elaborate later.

A *symbol* is a sign or a mark that can be distinguished from any other symbols. One often thinks of it as a letter in a word, or a digit in a number.
“Symbol” in other Languages

- Arabic: رمز (ar) m (ramz)
- Armenian: հատկանիշ (hy) (xorhdaniş), նշան (hy) (simvol)
- Chinese:
  Mandarin: 符号 (zh), 符号 (zh) (fùhào)
- Finnish: merkki (fi), symboli (fi)
- French: symbole (fr) m
- Georgian: სიმბოლო (simbolo)
- German: Symbol (de) n, Zeichen (de) n
- Greek: σύμβολο (el) n (sýmvolo)
  Ancient Greek: σύμβολον n (sûmbolon), σήμειον n (sêmeión)
- Norwegian:
  Bokmål: symbol (no) n
  Nynorsk: symbol (no) n
- Portuguese: símbolo (pt) m
- Romanian: simbol (ro) m
- Russian: символ (ru) m (sîmvol), знак (ru) m (znak)
- Spanish: símbolo (es) m
- Swedish: symbol (sv) c, tecken (sv) n
- Turkish: simge (tr), sembol (tr)
Dictionary Definitions of “Symbol”

1. A printed or written letter, figure, or other character or a combination of letters or the like used to designate something: the algebraic symbol $x$; the chemical symbol Au; a musical note ♪; the “recycle” symbol ♻. Synonyms: sign, glyph, mark, ideogram.

2. A thing that represents or stands for something else, especially a material object representing something immaterial or abstract. Synonyms: hallmark, emblem, token. Example: “The limousine was another symbol of his wealth and authority.” “A heart shape is the symbol of love.” “The wheel in the Indian flag is a symbol of peace.”

3. A shape or sign used to represent something such as an organization, e.g., a red cross or a Star of David. Synonyms: logo, trademark, insignia, icon. Examples: NPR, CBS, and KFC are no longer abbreviations; they are symbols of their respective organizations.
But here we use the word differently.

4. In the context of formal languages, a symbol is a printed or written letter, figure or the like which stands for nothing by itself and serves without interpretation as one indivisible element of a notational system.

A symbol is not to be confused with a name. For example, a variable is a name that stands for a location in a computer’s memory. We often use symbols for names to stand for something. A name or an identifier is a symbol, often composed of letters that names something. The name “mod” names the operation of taking the remainder of a quotient. For example, in genealogy the symbol ⊘ is used as a name to stand for “killed in action.”
The meaning or purpose of a symbol comes from a definition or customary usage or agreement and not from its form.

A symbol may appear complex and carry with it an implied message, for example:

But abstractly, one symbol is as good as another and one should never lose sight of the mathematical definitions that provide the true context.
However, the system of symbols (the notational system) might suggest how they are used. This serves as a hint to the reader. So a system of symbol forms $q_0, q_1, \ldots$, might be used as an aid to the reader by suggesting their origin or intended use. Compound symbols or symbols with micro-syntax are all used as an aid in communication. For example, a notational system formed as the cross product of two sets of symbols:

$$\langle q_0, p_0 \rangle, \langle q_0, p_1 \rangle, \langle q_1, p_1 \rangle, \ldots$$
In Sudoku puzzles the symbols do not stand for something. So the symbol 1 does not mean a cardinality of one, any more than a cheeseburger emoji does. And a cheeseburger emoji in a Sudoku puzzle does not mean food any more than the digit 1.
A Sudoku puzzle does not depend on the choice of symbols.
All those previous Sudoku puzzles were essentially the same as there was a consistent correspondence used between the sets of symbols. For that matter, any permutation of the symbols yields essentially the same puzzle, too.
Do symbols have names?
Can names denote symbols?
Can symbols have syntax?
“Symbols [atoms] are language constructs that start with a quote and continue with one or more letters”
Can anything be a symbol?
Formal language theory is indifferent to (or parameterized by) the underlying alphabet.

This is how distinguish phrase structure from lexical structure: same theory, different alphabets. (But different levels of the theory: regular and DFA, context-free and PDA.)
Alphabet

*Alphabet*. The *alphabet* is the finite set of symbols of indivisible elements making up the formal language. These are the basic, atomic, distinct elements.

\[
\Sigma = \{a, b, c, d\} \\
\Sigma = \{0, 1\} \\
\Sigma = \{a, b, \ldots, z, 0, 1, \ldots, 9\} \\
\Sigma = \text{Latin-1} \\
\Sigma = \text{Unicode} \\
\Sigma = \{\text{if, then, (, ), ', ', ...}\}
\]

Theoreticians like to use the set \(\Sigma = \{0, 1\}\) because it is so simple. The symbols might even be composed of other sub-atomic symbols. Important character sets for programming languages to use and be written in are Latin-1, Latin-0, and Unicode, not just US-ASCII.
Unicode Supports Many Scripts

Latin:  A  B  C  D  E  ...

Arabic:  ... ب ت ث ج ح خ ...

Hebrew:  ... א ב ג ד ה ...

Armenian:  Ա Բ Գ Դ Ե Զ Է Զ Է Ը Թ Ժ ...

Cyrillic:  А Б В Г Д ...

Devanagari:  क ह ग घ च छ ...

Thai:  ก ข ฃ ค ฅ ฆ ...
Strings. Given an alphabet $\Sigma = \{a, b, c, d\}$. A string is an (ordered) sequence of symbols from the alphabet. E.g., $aaa$, $bac$, $d$.

The sequence may contain no symbols, this empty string is given a special notation "". (Some authors use $\epsilon$, the Greek letter epsilon.)

The most important operation on strings is string concatenation which is sometimes denoted by the $\cdot$ operator. E.g., $ab \cdot db = abdb$, $da \cdot ba = daba$. 
More Strings

Given an alphabet $\Sigma = \{0, 1\}$, the following are strings: 0, $\epsilon$, 111, 010, 1.

Given an alphabet $\Sigma = \{a, b, \ldots, z\}$, the following are strings: $\epsilon$, x, aeiou, hello, biopsy.

Using Latin-1 as the alphabet, the following are strings: <Hola!, ¡Hola!, >Què pasa?, "Hi. Bye", größtergemeinsamer Teiler.

Using the alphabet $\Sigma$

$$\{\text{if, then, else, begin, end, (, ), ', '}\}$$

the following are strings:

1. else )
2. if then
3. begin begin , , end end
Formal Language

Definition: A formal language is a set of strings over an alphabet.
Significance: A formal language expresses information unambiguously.
Example: \{ab, bc, acc\}
Formal Language

*Formal language.* A *formal language* is a set of strings over some alphabet. Given an alphabet $\Sigma = \{0, 1\}$, the following are formal languages:

\[
\begin{align*}
L_0 & = \emptyset \\
L_1 & = \{1\} \\
L_2 & = \{1, 0101\} \\
L_3 & = \{0, 01, 00\} \\
L_4 & = \{1, 01, 11, 001, 011, 101, 111, \ldots\} \\
L_5 & = \{\epsilon, 1, 11, 111, 1111, \ldots\} \\
L_6 & = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\} = \Sigma^*
\end{align*}
\]

The collection of legal programs in a programming language is also an example of a formal language.
The mechanisms for describing formal languages can be categorized in a hierarchy according to their power. This hierarchy is known as the Chomsky hierarchy after the MIT linguist Noam Chomsky.
Chomsky Hierarchy
Chomsky Hierarchy

<table>
<thead>
<tr>
<th>name</th>
<th>machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 recursively enumerable</td>
<td>Turing machine</td>
</tr>
<tr>
<td>1 context-sensitive</td>
<td>linear-bounded automata</td>
</tr>
<tr>
<td>2 context-free</td>
<td>pushdown automata</td>
</tr>
<tr>
<td>3 regular</td>
<td>finite automata</td>
</tr>
</tbody>
</table>

- Regular expressions are used to describe tokens in programming languages
- BNF, CFG used to describe syntax of programming languages
- [Context-sensitive languages play no role in programming languages]
- attribute grammars, Post systems used in semantics
$U$ is the set of all formal languages.
Each dot is a formal language. $U$ is the set of formal languages: $U = P(\Sigma^*)$ where $\Sigma$ is the alphabet.
Description Mechanisms

1. Regular expressions (Appel, page 19, but not in Webber or Mitchell; briefly mentioned in Sebesta)
2. BNF (Chapters 2 and three of Webber)
Regular Expressions (Big Picture)

Regular Expressions

Finite Automata ← denote → Regular Languages

executable!
Regular Expressions (Literature)


Sams Teach Yourself Regular Expressions in 10 Minutes, 1st edition. by Ben Forta

Introducing Regular Expressions: Unraveling Regular Expressions, Step-by-Step

Regular Expressions: Simplicity and Power in Code by Douglas McLain Berdeaux

Whenever I learn a new skill I concoct elaborate fantasy scenarios where it lets me save the day.

Oh no! The killer must have followed her on vacation!

But to find them we'd have to search through 200 MB of emails looking for something formatted like an address!

It's hopeless!

Everybody stand back.

I know regular expressions.

xkcd—a webcomic of romance, sarcasm, math, and language by Randall Munroe
Regular Expressions

Some people, when confronted with a problem think “I know, I’ll use regular expressions.” Now they have two problems.

From a post by Jamie Zawinski to Usenet newsgroup alt.religion.emacs in 1997.
Regular Expressions (Syntax)

- What are they? (syntax)
- What do they mean? (semantics)
- Some examples of use with grep

For the class on compiler construction:
- Can a language denoted by a regular expression be effectively recognized? (Implementation)
- Can tokens in a programming language be effectively recognized? (Scanning)
1. Empty. $\emptyset$

2. Atom. Any single symbol of $a \in \Sigma$ is a regular expression.

3. Alternation. If $r_1$ is a regular expression and $r_2$ is a regular expression, then $(r_1 + r_2)$ is a regular expression.

4. Concatenation. If $r_1$ and $r_2$ are regular expressions, then $(r_1 \cdot r_2)$ is a regular expression.

5. Closure. If $r$ is a regular expression, then $(r)^*$ is a regular expression.
We omit parentheses following well-known rules:

- The outermost pair of parentheses is distracting.
- Alternation and concatenation can be considered left associative. (Semantically they are associative operators, so it does not make much difference.)
- Closure binds more tightly than concatenation which binds more tightly than alternation. (Similar to exponentiation, multiplication, and addition in traditional arithmetic expressions.)

Also, we sometimes omit the concatenation operator \( \cdot \) using mere juxtaposition to indicate concatenation.
Regular Expressions (Haskell)

\[
\text{data} \ \text{Regex} \ a = \text{Empty} \mid \text{Sym} \ a \mid \text{Star} \ (\text{Regex} \ a)
\quad \text{Alt} \ (\text{Regex} \ a) \ (\text{Regex} \ a) \mid \text{Concat} \ (\text{Regex} \ a) \ (\text{Regex} \ a)
\quad \text{deriving} \ (\text{Eq}, \ \text{Ord})
\]

\text{Alt} \ (\text{Sym} \ 'a', \ \text{Concat} \ (\text{Sym} \ 'b', \ \text{Sym} \ 'c')) \quad -- \quad a+(bc)
\text{Concat} \ (\text{Alt} \ (\text{Sym} \ 'a', \ \text{Sym} \ 'b'), \ \text{Sym} \ 'c') \quad -- \quad (a+b)c
Regular Expressions (Examples)

\[ a \cdot b \]
\[ a + b \]
\[ a \cdot (b + c) \]
\[ a + b \cdot c \]
\[ a^* \]
\[ a + (b \cdot c)^* \]
\[ (a + (b \cdot c))^* \]

(using the alphabet \[ \Sigma = \{a, b, c\} \])
Regular Expressions (Examples)

(Omitting the “centered dot” and using the | for alternation.)

\[
\begin{align*}
ab \\
a \mid b \\
a(b \mid c) \\
a \mid bc \\
a^* \\
a \mid (bc)^* \\
(a \mid (bc))^*
\end{align*}
\]

(using the alphabet \( \Sigma = \{a, b, c\} \))
Regular Expressions (Examples)

01
101
1 + 0
1(0 + 1)
0 + 10
0*
1 + (01)*
(0 + (10)*
(0 + (10)*)

(using the alphabet $\Sigma = \{0, 1\}$ and omitting the $\cdot$ symbol)
Regular Expressions (Semantics)

That is what regular expressions look like (syntax). What do regular expressions mean (semantics)?
Each regular expression denotes a formal language (a set of strings).
There are an infinite number of regular expressions. (Each one is finitely constructed.)
Just like programs in programming languages.
We must find a way to define the meaning or denotation of each regular expression.
So, we define a (recursive) function from regular expressions (as pieces of syntax) to sets of strings (languages).
1. Empty. The language with no strings.
2. Atom. The language with one string of length one—$a$ itself. Note that the notation $a$ is ambiguous. It stands for both the symbol and the language.
3. Alternation. $(r_1 + r_2)$ is the union of two languages.
4. Concatenation. $(r_1 \cdot r_2)$ is the set all strings beginning in one language and then followed by a string in the second.
5. Closure. $(r)^*$ is zero, one, or more strings.
Regular Expressions (Examples)

\[ a \cdot b = \{ab\} \]
\[ a + b = \{a, b\} \]
\[ a \cdot (b + c) = \{ab, ac\} \]
\[ a + (b \cdot c) = \{a, bc\} \]
\[ a^* = \{\epsilon, a, aa, aaa, \ldots\} \]
\[ a + (b \cdot c)^* = \{\epsilon, a, bc, bcbc, \ldots\} \]
\[ (a + (b \cdot c))^* = \{\epsilon, a, bc, aa, abc, bcbc, \ldots\} \]

(using the alphabet \( \Sigma = \{a, b, c\} \))
Some authors replace one of the five cases of the definition

1. Empty. $\emptyset$ denoting the language with no strings

with another case

1. Epsilon. $\epsilon$ denoting the set consisting of the single string ""

Do you see the difference?

Notice the $\epsilon$ is superfluous as $\{\epsilon\}$ is represented by $\emptyset^*$. Can you prove this?
Regular Expressions (Formal Semantics)

A regular expression denotes a formal language (set of strings) over an alphabet $A$ by means of a function $D$. The function $D$ takes a regular expression and associates with it a particular formal language. The function is defined recursively over the five cases of the inductive definition of the set of regular expressions.

1. Empty.

$$D[\emptyset] = \{\}$$

2. Atom. For each $a \in \Sigma$,

$$D[a] = \{a\}$$

3. Alternation.

$$D[(r_1 + r_2)] = D[r_1] \cup D[r_2]$$
Regular Expressions (Formal Semantics)


\[ D[(r_1 \cdot r_2)] = \{ x \cdot y \mid x \in D[r_1], y \in D[r_2] \} \]

where \( x \cdot y \) is string concatenation.

5. Closure.

\[ D[(r)^*] = \bigcup_i (D[r])^i \]

where \( S^i \) is defined recursively as follows:

\[ S^0 = \{ \epsilon \} \]
\[ S^{i+1} = \{ x \cdot y \mid x \in S, y \in S^i \} \]
Using the Definitions

\[
\mathcal{D}[(a \cdot b)] = \{x \cdot y \mid x \in \mathcal{D}[a], y \in \mathcal{D}[b]\}
\]
\[
= \{x \cdot y \mid x \in \{a\}, y \in \{b\}\}
\]
\[
= \{a \cdot b\} = \{ab\}
\]

\[
\mathcal{D}[(a + (a \cdot b))] = \mathcal{D}[a] \cup \mathcal{D}[(a \cdot b)]
\]
\[
= \{a\} \cup \{ab\}
\]
\[
= \{a, ab\}
\]
Using the Definitions

\[
\mathcal{D}[a + (b \cdot c)]^* = \bigcup_i \mathcal{D}[a + (b \cdot c)]^i
\]

\[
= \{\epsilon\} \cup \{a, bc\} \cup \mathcal{D}[a + (b \cdot c)]^2 \cup \ldots
\]

\[
= \{\epsilon, a, bc\} \cup \{aa, abc, bca, bcbc\} \cup \ldots
\]

\[
= \{\epsilon, a, aa, bc, abc, bca, bcbc\} \cup \ldots
\]

\[
= \{\epsilon, a, aa, bc, aaa, abc, bca, aabc, abca, bcaa, bcba, \ldots\}
\]
Digression: Induction

Is the recursively defined function $D$ well-defined? Yes.
When are such recursively defined functions well-defined? Over free-generated sets.
Digression: Induction

Consider the following inductive definition of a subset $M$ of $\text{Nat}$, the natural numbers, using integer multiplication $\cdot$:

- $1 \in M$,
- if $n \in M$, then $9 \cdot n \in M$,
- if $n \in M$, then $23 \cdot n \in M$.

Suppose we define the function $g : M \rightarrow \text{Nat}$ inductively by:

- $g(1) = 1$,
- $g(9 \cdot n) = 9$,
- $g(23 \cdot n) = 23$.

Prove that $0 = 1!$
To make regular expressions more convenient, regular expressions are almost always extended with new notation. Here are some additional meta-symbols commonly seen (perhaps with different syntax). Regular expressions with these new meta-symbols can be defined in terms of the original definitions. Let $r$ be a regular expression over the alphabet $\Sigma$.

1. Optional. $r? = (r + \emptyset^*)$

2. One or more. (This is a second and conflicting use of the meta-character +.)
   
   $r^+ = (r \cdot r^*)$

3. Any. $.= (a + b + \ldots + y + z)$ where $\Sigma = a, b, \ldots, y, z$.

4. Range. $[a – z] = (a + b + \ldots + y + z)$. (Assumes that $\Sigma$ is ordered.)

5. Range complement. $[\neg c – x] = (a + b + y + z)$. (Assumes that $\Sigma$ is ordered.)
Where are regular expressions used?
The program `grep` is a command-line utility for searching text originally written for Unix. The `grep` command searches text files for lines matching a given regular expression and prints matching lines to the program's standard output.
Suppose the file preamble.txt has the following lines:

We the People of the United States, in Order to form a more perfect Union, establish Justice, insure domestic Tranquility, provide for the common defence, promote the general Welfare, and secure the Blessings of Liberty to ourselves and our Posterity, do ordain and establish this Constitution for the United States of America.

```bash
> grep and preamble.txt
common defence, promote the general Welfare, and secure the Blessings of Liberty to ourselves and our Posterity, do ordain and establish

> grep -i people preamble.txt
We the People of the United States, in Order to form a more perfect
of Liberty to ourselves and our Posterity, do ordain and establish

(But does not match "domestic.")

common defence, promote the general Welfare, and secure the Blessings
Practical Regular Expressions

grep options regexp filename

Options:

-E extended regular expression
-i ignore case
-x match whole line
-w match word in line

Syntax of regular expressions for grep

| or | $ | end of line |
| .  | ? | optional    |
| [] | {n,m} | n through m times |
[^ ] set compliment () grouping
grep

Words one can spell with hex “digits.”

grep -E -w -i ’[a-f]{3,4}’ /usr/dict/words
grep -E -w -i ’[a-f]{3,4}’ /usr/share/dict/words

among others. (Details: “beef” and “beef’s”; “cafe” versus “café”.)
grep

Words one can spell with hex “digits.”

grep -E -w -i ‘[a-f]{3,4}’ /usr/dict/words

grep -E -w -i ‘[a-f]{3,4}’ /usr/share/dict/words

babe
beef
cafe
deaf
deaf
dead
dead
fade
feed

among others. (Details: “beef” and “beef’s”; “cafe” versus “café”.)
grep

grep -E '.{5,}' /usr/dict/words
grep

grep -E '.{5,}' /usr/dict/words

Aarhus
Aaron
Ababa
aback
abacus
...
zombie
zoology
Zoroastrian
zucchini
Zurich
zygote
grep

grep -i -E '[aeiou]{4,}' /usr/dict/words
grep

grep -i -E '[aeiou]{4,}' /usr/dict/words

aqueous
Hawaiian
IEEE
obsequious
onomatopoeia
pharmacopoeia
prosopopoeia
queue
Sequoia
grep

grep -i -E '(q[^u]|q$)' /usr/dict/words
grep

grep -i -E '(q[^u]|q$)' /usr/dict/words

CEQ
Colloq
IQ
Iraq
q
Qatar
QED
q’s
seq
grep

Is there a regular expression that matches those words whose letters appear in alphabetical order?

grep -x -E <regular expression> /usr/dict/words
Is there a regular expression that matches those words whose letters appear in alphabetical order?

```
grep -x -E <regular expression> /usr/dict/words

```
grep

Is there a regular expression that matches those words whose letters appear in alphabetical order?

grep -x -E <regular expression> /usr/dict/words

/usr/dict/words

almost
begin
below
biopsy
dirty
empty
first
glory

(Some of the longer words.)
Can we modify the previous example to allow double letters?

grep -x -E 'a*b*c*d*e*f*g*h*i*j*k*l*m*n*o*p*q*r*s*t*u*v*w*x*y*z*
/usr/dict/words

accent
almost
biopsy
choosy
effort
floppy
glossy
knotty

(Some of the longer words.)
grep

grep -E -e '(y.*){3,}' /usr/dict/wordsA
grep -E -e '(y.*){3,}' /usr/dict/wordsA

The words with at least three y’s.

polytypy chromosomal variation between populations psychophysiology the way mind and body interact synonymy the state of being synonymous syzygy straight-line alignment of 3 celestial bodies
grep -E -e '(.)\1\1' /usr/dict/words
grep -E -e '(.))1\1' /usr/dict/words

AAA
AAAS
IEEE
iii
viii
Perl supports the usual regular expressions:

\                 Quote the next metacharacter
^                 Match the beginning of the line
.                 Match any character (except newline)
$                 Match the end of the line (or before newline at the end)
|                 Alternation
(..)              Grouping
[..]              Character class

*                 Match 0 or more times
+                 Match 1 or more times
?                 Match 1 or 0 times
{n}               Match exactly n times
{n,}              Match at least n times
{n,m}             Match at least n but not more than m times
PERL supports “reluctant” and “possessive” quantifiers in addition to the “greedy” quantifiers. PERL has additional boundary matchers, Unicode character class operators, and look-ahead and look-behind matchers. Some of these require exponential back-tracking.

\p{property}    Match a character with Unicode property
\P{property}    Match a character without Unicode property
(?=pattern)     A zero-width positive look-ahead assertion
(?!pattern)     A zero-width negative look-ahead assertion
(?<=pattern)    A zero-width positive look-behind assertion
(?<!pattern)    A zero-width negative look-behind assertion

For more information see the documentation at www.perldoc.com.
Java supports regular expression in the class `java.util.regex.Pattern`. It has character class operations for union and intersection.

- `[…]` Character class
- `[^...]` Complement
- `[[...]]` Union
- `[[...]]` Intersection
- `[[...]]` Substraction
// Greedy quantifiers
String match = find("A.*c", "AbcAbc");  // AbcAbc
match = find("A.+", "AbcAbc");  // AbcAbc

// Nongreedy quantifiers
match = find("A.*?c", "AbcAbc");  // Abc
match = find("A.+?", "AbcAbc");  // Ab

// Find first substring in input that matches pattern.
static String find (String pat, CharSequence input) {
    final Pattern pattern = Pattern.compile(pat);
    final Matcher matcher = pattern.matcher(input);
    if (matcher.find()) {
        return matcher.group();
    }
    return null;
}
Using regular expressions to match comments often runs into two problems:

- The “every character” pattern often does not match the characters indicating a newline.

  /* The first line,
   but not the second line. */

- The greedy “star” will match more than what was intended.

  x = 1; /* Both comments */
  x = 2;
  /* at once! */ x = 3;

```java
final static String commentMultilineString = "/*/.*?\*/";
final static Pattern multiLineCommentPattern =
    Pattern.compile (commentMultilineString, Pattern.DOTALL);
```
Tokens

Regular expressions are used in the definition of programming languages to describe the basic lexical units—tokens.

1. NUMBER: \[0 - 9\]+  
2. IDENTIFIER: \[a - zA - Z][a - zA - Z0 - 9]*  
3. a keyword: `begin`  
4. a hex number: `0x[0 - 9A - F]+`  
5. REAL: \[0 - 9\] +\.'[0 - 9] + (E[0 - 9]+)?

Metacharacters: [] +’ -()?|
Token Stream

with Identifier ; procedure

-- comment begin Identifier :=

3.14159 Identifier ....
Given a program

```c
float match0 (char *s) /* find a zero */
{ if (!strncmp (s, "0.0", 3))
   return 0.;
}
```

the lexical analyzer will return the stream

```
FLOAT ID(MATCH0) LPAREN CHAR STAR ID(s) RPAREN LBRACE BANG ID(STRNCMP) LPAREN ID(s) COMMA STRING(0.0) COMMA NUM(3) RPAREN RPAREN RETURN REAL(0.0) SEMI RBRACE EOF
```
Scanning or tokenization described in words:

An identifier is a sequence of letters and digits; the first character must be a letter. The underscore _ counts as a letter. Upper- and lowercase letters are different. If the input stream has been parsed into tokens up to a given character, the next token is taken to include the longest string of characters that could possibly constitute a token. Blanks, tabs, newlines, and comments are ignored except as they serve to separate tokens. Some white space is required to separate otherwise adjacent identifiers, keywords, and constants.
Regular expressions

1. How do you recognize if a string matches a regular expression?
2. How do you tokenize a character string?

These questions are left to a compiler construction class.
Expressiveness of RE

We like RE because they describe strings.
RE are great: easily understandable, concise, recognizers are easy to make, . . .
But,
Expressiveness of RE

We like RE because they describe strings.
RE are great: easily understandable, concise, recognizers are easy to make, . . .
But,
The major shortcoming of regular expressions, as far as the description of programming languages is concerned, is that bracketing is not expressible. Bracketing is indispensable. For example, arithmetic expressions require opening and closing parentheses. Sequences of statements are often bracketed by \texttt{begin/end} pairs.

The set of strings

$$\{a^n \cdot b^n\} = \{\epsilon, ab, aabb, aaabbb, \ldots\}$$

is not regular!
Confext Free Grammars
Backus-Naur Form
John Backus proposed a metalanguage of “metalinguistic formulas" to describe syntax of the new programming language ALGOL. Apparently, Backus was familiar with Chomsky’s work.

Peter Naur called Backus’s notation *Backus normal form*. Soon afterward, Donald Knuth argued that BNF should rather be read as Backus-Naur form, as it is “not a normal form in the conventional sense” (CACM, 1964), unlike, for instance, Chomsky normal form.
In his book *Computing: A Human Activity* (1992), Naur rejected the formalist view of programming as a branch of mathematics. He did not like being associated with the Backus-Naur Form (a notation technique for context-free grammars attributed to him by Donald Knuth), and said he would prefer it to be called the Backus Normal Form.

Naur disliked the term “computer science,” suggesting it instead be called “datalogy” or “data science.” The term “dataology” has been adopted in Denmark and in Sweden as *datalogi*, while “data science” is used to refer to data analysis (as in statistics and databases).
BNF (Backus-Naur Form)

Conventions (e.g., tokens in bold font) and meta-symbols:

::= “is defined as”
| “or”
[] “optional” (not necessary, EBNF)
{} “zero or more” (not necessary, EBNF)

BNF is a collection of rules or productions. Each rule has a LHS and a RHS. The LHS is a syntactic category (a part of the language), called a *nonterminal* and the RHS is a sequence of nonterminals and tokens (called *terminals*).

\[
LHS_1 ::= RHS_1 \\
LHS_2 ::= RHS_2 \\
\vdots \\
LHS_n ::= RHS_n
\]
Examples and Derivations

Grammar 1 and derivation: “revolutionary ideas”
Example 3.1, page 129. Program with statement lists
Example 3.2, page 131. Simple assignment statements
Sebesta, seventh edition
Example Grammar

sentence ::= subject predicate .
subject ::= article { adjective } noun
predicate ::= verb [ adverb ]
article ::= A | The
adjective ::= green | colorless | big | new | revolutionary
noun ::= caterpillar | idea
verb ::= sleeps | appears | crawls
adverb ::= furiously | infrequently
**Derivation**

*Sentential form.* A sentential form is a string of terminal and nonterminal symbols.

*Derivation.* A derivation of a sentential form is created from a nonterminal by repeatedly replacing the LHS nonterminal of some rule with its RHS. Every BNF definition is a description of a formal language (a set of string), the language of all sentences derivable from a syntactic category.

We derive two sentences used as example by Noam Chomsky—one sentence is meaningful, the other makes no sense.
Example Derivation

sentence ⇒

A new revolutionary idea appears infrequently.
Example Derivation

\[ \text{sentence} \Rightarrow \]
\[ \text{subject predicate} . \Rightarrow \]
Example Derivation

\[
\text{sentence} \Rightarrow \\
\text{subject predicate} \Rightarrow \\
\text{article adjective adjective noun predicate} \Rightarrow
\]
Example Derivation

\[
\text{sentence } \Rightarrow \\
\text{subject predicate } \Rightarrow \\
\text{article adjective adjective noun predicate } \Rightarrow \\
A \text{ adjective adjective noun predicate } \Rightarrow
\]
Example Derivation

sentence⇒
subject predicate . ⇒
article adjective adjective noun predicate . ⇒
A adjective adjective noun predicate . ⇒
A new adjective noun predicate . ⇒
Example Derivation

\[
\text{sentence} \Rightarrow \\
\text{subject predicate} . \Rightarrow \\
\text{article adjective adjective noun predicate} . \Rightarrow \\
\text{A adjective adjective noun predicate} . \Rightarrow \\
\text{A new adjective noun predicate} . \Rightarrow \\
\text{A new revolutionary noun predicate} . \Rightarrow 
\]
Example Derivation

sentence ⇒
subject predicate . ⇒
article adjective adjective noun predicate . ⇒
A adjective adjective noun predicate . ⇒
A new adjective noun predicate . ⇒
A new revolutionary noun predicate . ⇒
A new revolutionary idea predicate . ⇒
Example Derivation

sentence ⇒
subject predicate . ⇒
article adjective adjective noun predicate . ⇒
A adjective adjective noun predicate . ⇒
A new adjective noun predicate . ⇒
A new revolutionary noun predicate . ⇒
A new revolutionary idea predicate . ⇒
A new revolutionary idea verb adverb . ⇒
Example Derivation

sentence ⇒

subject predicate . ⇒

article adjective adjective noun predicate . ⇒

A adjective adjective noun predicate . ⇒

A new adjective noun predicate . ⇒

A new revolutionary noun predicate . ⇒

A new revolutionary idea predicate . ⇒

A new revolutionary idea verb adverb . ⇒

A new revolutionary idea appears adverb . ⇒
Example Derivation

sentence $\Rightarrow$

subject predicate . $\Rightarrow$

article adjective adjective noun predicate . $\Rightarrow$

A adjective adjective noun predicate . $\Rightarrow$

A new adjective noun predicate . $\Rightarrow$

A new revolutionary noun predicate . $\Rightarrow$

A new revolutionary idea predicate . $\Rightarrow$

A new revolutionary idea verb adverb . $\Rightarrow$

A new revolutionary idea appears adverb . $\Rightarrow$

A new revolutionary idea appears infrequently .
Example Derivation

sentence⇒
subject predicate . ⇒
article adjective adjective noun predicate . ⇒
A adjective adjective noun predicate . ⇒
A colorless adjective noun predicate . ⇒
A colorless green noun predicate . ⇒
A colorless green idea predicate . ⇒
Example Derivation

```
sentence \Rightarrow
  subject \; predicate \; . \Rightarrow
  article \; adjective \; adjective \; noun \; predicate \; . \Rightarrow
  A \; adjective \; adjective \; noun \; predicate \; . \Rightarrow
  A \; colorless \; adjective \; noun \; predicate \; . \Rightarrow
  A \; colorless \; green \; noun \; predicate \; . \Rightarrow
  A \; colorless \; green \; idea \; predicate \; . \Rightarrow
  A \; colorless \; green \; idea \; verb \; adverb \; . \Rightarrow
```
Example Derivation

\[
\text{sentence} \Rightarrow \\
\text{subject predicate} \Rightarrow \\
\text{article adjective adjective noun predicate} \Rightarrow \\
A \text{ adjective adjective noun predicate} \Rightarrow \\
A \text{ colorless adjective noun predicate} \Rightarrow \\
A \text{ colorless green noun predicate} \Rightarrow \\
A \text{ colorless green idea predicate} \Rightarrow \\
A \text{ colorless green idea verb adverb} \Rightarrow \\
A \text{ colorless green idea sleeps adverb} \Rightarrow 
\]
Example Derivation

sentence ⇒
  subject predicate . ⇒
  article adjective adjective noun predicate . ⇒
  A adjective adjective noun predicate . ⇒
  A colorless adjective noun predicate . ⇒
  A colorless green noun predicate . ⇒
  A colorless green idea predicate . ⇒
  A colorless green idea verb adverb . ⇒
  A colorless green idea sleeps adverb . ⇒
  A colorless green idea sleeps furiously .
Syntax versus Semantics

Two sentences (with the same structure, syntax) derived from the grammar.

- A new revolutionary idea appears infrequently.
- A colorless green idea sleeps furiously.

One sentence is meaningful, the other is meaningless. Clearly, the meaning of the words (semantics) is important.
Examples and Derivations

* Grammar 1 and derivation: “revolutionary ideas”
Example 3.1, page 112. Program with statement lists
Example 3.2, page 113. Simple assignment statements
Sebesta, fifth edition
Alternate Styles of BNF

“traditional” style:
\langle traditional style \rangle ::= \textbf{key}
Nonterminals in angle brackets, terminals in bold font.

CFG or “mathematical” style:
\[ A \rightarrow B a \]
Nonterminals in uppercase letters, terminal in lower case letters.

“verbatim” style:
verbatim_style ::= "key"

ISO 14977 is much like verbatim style.

Wirth, “What can we do about the unnecessary diversity of notation for syntactic definitions?” CACM, volume 20, number 11, November 1977, page 822.
In addition to linear forms of BNF there are also syntax graphs, also known as syntax diagrams.
See Chapter 2, page 21 of Webber
Figure 3.6, page 122 of Sebesta, fifth edition (not in sixth edition).
See Hyper GOS syntax diagram generator.
Syntax Graphs

if_statement ::= 
  "if" condition "then"
sequence_of_statements
{ "elsif" condition "then"
sequence_of_statements }
[ "else"
sequence_of_statements ]
"end" "if" ";;"
A grammar is a 4-tuple $\langle T, N, P, S \rangle$

- $T$ is the set of terminal symbols,
- $N$ set of nonterminal symbols, $T \cap N = \emptyset$,
- $S$, a nonterminal, is the start symbol,
- $P$ are the productions of the grammar.

A production has the form $\alpha \rightarrow \beta$ where $\alpha$ and $\beta$ are strings of terminals and nonterminals ($\alpha$ can’t be the empty string). We are primarily interested in context-free grammars in which $\alpha$ is a single nonterminal.
Example 1

Let $G$ be the grammar defined by the following productions.

\[
S \rightarrow A \ B \\
A \rightarrow \epsilon \\
A \rightarrow A \ a \\
B \rightarrow \epsilon \\
B \rightarrow B \ b
\]

(We infer easily that $T$, the set of terminals, is $\{a, b\}$; $N$, the set of nonterminals, is $\{A, B\}$; and $S$ is the start symbol.)

\[
L_G = \{a^i \ b^j\}
\]
Example 2

Let $G$ be the grammar defined by the following productions.

\[
S \rightarrow \epsilon \\
S \rightarrow aSb
\]

(We infer easily that $T$, the set of terminals, is \{a, b\}; $N$, the set of nonterminals, is \{S\}; and $S$ is the start symbol.)

\[
L_G = \{a^i b^i\}
\]
A grammar is a description of a language. Every grammar denotes a formal language (set of strings). A string of nonterminals $\omega$ is in the language, if it is derivable from the start the symbol. We say $\gamma \alpha \delta \Rightarrow \gamma \beta \delta$ (derives in one step) whenever $\alpha \rightarrow \beta$ is a production. We use $\alpha \Rightarrow^* \beta$ as the reflexive and transitive closure of the $\Rightarrow$ relation. A string $\omega$ of terminals is in the language defined by grammar $G$, if $S \Rightarrow^* \omega$. So, $L(G) = \{ \omega \in T^* \mid S \Rightarrow^* \omega \}$. In this case the string $\omega$ is called a sentence of $G$. If $S \Rightarrow^* \alpha$ where $\alpha$ may contain nonterminals, then we say $\alpha$ is a sentential form of the grammar $G$. 
**Chomsky Hierarchy**

<table>
<thead>
<tr>
<th>name</th>
<th>grammar</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 unrestricted</td>
<td></td>
</tr>
<tr>
<td>1 context-sensitive</td>
<td>$</td>
</tr>
<tr>
<td>2 context-free</td>
<td>$\alpha \in N$</td>
</tr>
<tr>
<td>3 regular</td>
<td>$A \rightarrow \omega B$, $A \rightarrow \omega$</td>
</tr>
</tbody>
</table>
Context-free grammar is the same as BNF.

\[
S ::= AB \\
A ::= A \left| \varepsilon \right. \\
B ::= B \left| \varepsilon \right. \\
\]

OR

\[
S ::= AB \\
A ::= Aa \left| \varepsilon \right. \\
B ::= Bb \left| \varepsilon \right. \\
\]
A formal definition of a parse tree for a context-free grammar is possible, but it is not illuminating. But the trees in the figures are suggestive. Each production used in the derivation of a string appears as a subtree in the diagram. The left-hand-side nonterminal appears as a node, and all the grammar symbols in the right-hand side of the production appear as children of this node.
An example form Sebesta, 7th, Example 3.2, page 131.
An grammar for simple assignment statement

\[
\begin{align*}
assign & ::= id := expr \\
expr & ::= expr + expr \\
& | expr * expr \\
& | ( expr ) \\
& | id \\
id & ::= A \mid B \mid C
\end{align*}
\]
stmt ::= if-stmt | while-stmt | begin-stmt | asgn-stmt
if-stmt ::= if bool-exp then stmt else stmt
while-stmt ::= while bool-exp do stmt
begin-stmt ::= begin stmt-list end
asgn-stmt ::= variable := arith-exp
stmt-list ::= stmt | stmt ; stmt
bool-exp ::= arith-exp compare arith-exp
compare-op ::= < | <= | > | >= | =
arith-exp ::= variable | const | arith-exp op arith-exp
arith-op ::= + | * | − | /
variable ::= a | b | c | x | y | z
constant ::= 0 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9
stmt ::= if-stmt | while-stmt | begin-stmt | asgn-stmt ;
if-stmt ::= "if" bool-expr "then" stmt "else" stmt ;
if-stmt ::= "if" bool-expr "then" stmt ;
while-stmt ::= "while" bool-expr "do" stmt ;
begin-stmt ::= "begin" stmt-list "end" ;
asgn-stmt ::= variable ":=" arith-expr ;
stmt-list ::= stmt | <stmt> ";",' <stmt-list> ;
bool-expr ::= arith-expr compare-op arith-expr ;
compare-op ::= ":<" | ":=" | ":opper" | ":>=" | ":=" | ":<>" ;
arith-expr ::= variable | constant | arith-expr arith-op arith-expr ;
arith-op ::= ":+" | ":-" | ":*" | ":/" ;
variable ::= "a" | "b" | "c" | "c" | "x" | "x" | "z" ;
Example Derivation

\[
\text{assign} \Rightarrow
\]

\[
A := \text{expr} \Rightarrow A := \text{expr} \ast \text{expr} \Rightarrow A := \text{id} \ast \text{expr} \Rightarrow A := B \ast \text{expr} \Rightarrow A := B \ast (\text{expr}) \Rightarrow A := B \ast (A + \text{expr}) \Rightarrow A := B \ast (A + \text{id}) \Rightarrow A := B \ast (A + C)
\]
Example Derivation

assign ⇒
    id := expr ⇒
Example Derivation

\[\text{assign} \Rightarrow \]
\[id := expr \Rightarrow \]
\[A := expr \Rightarrow \]
Example Derivation

assign \Rightarrow
\begin{align*}
id & := expr \Rightarrow \\
A & := expr \Rightarrow \\
A & := expr \ast expr \Rightarrow
\end{align*}
Example Derivation

assign ⇒

\[ id := expr \] ⇒
\[ A := expr \] ⇒
\[ A := expr \ast expr \] ⇒
\[ A := id \ast expr \] ⇒
\[ A := B \ast (expr) \] ⇒
\[ A := B \ast (id + expr) \] ⇒
\[ A := B \ast (A + expr) \] ⇒
\[ A := B \ast (A + id) \] ⇒
\[ A := B \ast (A + C) \]
Example Derivation

\[
\text{assign} \Rightarrow \\
id := \text{expr} \Rightarrow \\
A := \text{expr} \Rightarrow \\
A := \text{expr} \ast \text{expr} \Rightarrow \\
A := \text{id} \ast \text{expr} \Rightarrow \\
A := B \ast \text{expr} \Rightarrow \\
\text{A} := \text{B} \ast \text{expr} \Rightarrow \\
\]
Example Derivation

\[
\text{assign } \Rightarrow \\
\quad id := \text{expr } \Rightarrow \\
\quad A := \text{expr } \Rightarrow \\
\quad A := \text{expr } \ast \text{expr } \Rightarrow \\
\quad A := id \ast \text{expr } \Rightarrow \\
\quad A := B \ast \text{expr } \Rightarrow \\
\quad A := B \ast (\text{expr}) \Rightarrow 
\]
Example Derivation

\[
\text{assign } \Rightarrow \\
\quad id := expr \Rightarrow \\
\quad A := expr \Rightarrow \\
\quad A := expr \ast expr \Rightarrow \\
\quad A := id \ast expr \Rightarrow \\
\quad A := B \ast expr \Rightarrow \\
\quad A := B \ast ( expr ) \Rightarrow \\
\quad A := B \ast ( id + expr ) \Rightarrow
\]
Example Derivation

```
assign ⇒
  id := expr ⇒
A := expr ⇒
A := expr * expr ⇒
A := id * expr ⇒
A := B * expr ⇒
A := B * ( expr ) ⇒
A := B * ( id + expr ) ⇒
A := B * ( A + expr ) ⇒
```
Example Derivation

\[
\text{assign} \Rightarrow \\
\text{id} := \text{expr} \Rightarrow \\
\text{A} := \text{expr} \Rightarrow \\
\text{A} := \text{expr} \ast \text{expr} \Rightarrow \\
\text{A} := \text{id} \ast \text{expr} \Rightarrow \\
\text{A} := \text{B} \ast \text{expr} \Rightarrow \\
\text{A} := \text{B} \ast (\text{expr}) \Rightarrow \\
\text{A} := \text{B} \ast (\text{id} + \text{expr}) \Rightarrow \\
\text{A} := \text{B} \ast (\text{A} + \text{expr}) \Rightarrow \\
\text{A} := \text{B} \ast (\text{A} + \text{id}) \Rightarrow 
\]
Example Derivation

assign ⇒

\[ id := expr \Rightarrow \]
\[ A := expr \Rightarrow \]
\[ A := expr * expr \Rightarrow \]
\[ A := id * expr \Rightarrow \]
\[ A := B * expr \Rightarrow \]
\[ A := B * ( expr ) \Rightarrow \]
\[ A := B * ( id + expr ) \Rightarrow \]
\[ A := B * ( A + expr ) \Rightarrow \]
\[ A := B * ( A + id ) \Rightarrow \]
\[ A := B * ( A + C ) \]
Example Derivation

assign ⇒
Example Derivation

assign ⇒
id := expr ⇒
Example Derivation

\[
\begin{align*}
assign &\Rightarrow \\
id &:= expr \Rightarrow \\
A &:= expr \Rightarrow \\
\end{align*}
\]
Example Derivation

assign ⇒
  id := expr ⇒
  A := expr ⇒
  A := expr * expr ⇒

A := B * expr + expr ⇒
A := B * A + id ⇒
A := B * A + C
Example Derivation

assign ⇒
  id := expr ⇒
  A := expr ⇒
  A := expr ∗ expr ⇒
  A := id ∗ expr ⇒

A := B ∗ A + expr ⇒
A := B ∗ A + C
Example Derivation

assign \Rightarrow 
  id := expr \Rightarrow 
  A := expr \Rightarrow 
  A := expr * expr \Rightarrow 
  A := id * expr \Rightarrow 
  A := B * expr \Rightarrow
Example Derivation

\[\text{assign} \Rightarrow \]
\[id := expr \Rightarrow \]
\[A := expr \Rightarrow \]
\[A := expr \ast expr \Rightarrow \]
\[A := id \ast expr \Rightarrow \]
\[A := B \ast expr \Rightarrow \]
\[A := B \ast expr + expr \Rightarrow \]
Example Derivation

\[
\begin{align*}
\text{assign} & \Rightarrow \\
\text{id} & \Rightarrow expr \\
A & \Rightarrow expr \\
A & \Rightarrow expr \ast expr \\
B & \Rightarrow id \ast expr \\
B & \Rightarrow B \ast expr \\
B & \Rightarrow B \ast expr + expr \\
B & \Rightarrow B \ast id + expr
\end{align*}
\]
Example Derivation

assign ⇒
  id := expr ⇒
A := expr ⇒
A := expr * expr ⇒
A := id * expr ⇒
A := B * expr ⇒
A := B * expr + expr ⇒
A := B * id + expr ⇒
A := B * A + expr ⇒
Example Derivation

\[ \text{assign} \implies \]
\[ id := \text{expr} \implies \]
\[ A := \text{expr} \implies \]
\[ A := \text{expr} \cdot \text{expr} \implies \]
\[ A := \text{id} \cdot \text{expr} \implies \]
\[ A := B \cdot \text{expr} \implies \]
\[ A := B \cdot \text{expr} + \text{expr} \implies \]
\[ A := B \cdot \text{id} + \text{expr} \implies \]
\[ A := B \cdot A + \text{expr} \implies \]
\[ A := B \cdot A + \text{id} \implies \]
Example Derivation

assign ⇒
   id := expr ⇒
   A := expr ⇒
   A := expr * expr ⇒
   A := id * expr ⇒
   A := B * expr ⇒
   A := B * expr + expr ⇒
   A := B * id + expr ⇒
   A := B * A + expr ⇒
   A := B * A + id ⇒
   A := B * A + C
Example Derivation

assign ⇒
Example Derivation

assign ⇒

id := expr ⇒
Example Derivation

\begin{align*}
\text{assign} & \Rightarrow \\
\text{id} & := \text{expr} \Rightarrow \\
\text{A} & := \text{expr} \Rightarrow 
\end{align*}
Example Derivation

\[
\begin{align*}
assign & \Rightarrow \\
id & := expr \Rightarrow \\
A & := expr \Rightarrow \\
A & := expr + expr \Rightarrow
\end{align*}
\]
Example Derivation

assign ⇒
  id := expr ⇒
  A := expr ⇒
  A := expr + expr ⇒
  A := expr * expr + expr ⇒
Example Derivation

\[
\text{assign} \Rightarrow \\
\quad id := expr \Rightarrow \\
\quad A := expr \Rightarrow \\
\quad A := expr + expr \Rightarrow \\
\quad A := expr \ast expr + expr \Rightarrow \\
\quad A := id \ast expr + expr \Rightarrow 
\]
Example Derivation

\[
\text{assign} \Rightarrow \\
id := \text{expr} \Rightarrow \\
A := \text{expr} \Rightarrow \\
A := \text{expr} + \text{expr} \Rightarrow \\
A := \text{expr} \ast \text{expr} + \text{expr} \Rightarrow \\
A := \text{id} \ast \text{expr} + \text{expr} \Rightarrow \\
A := B \ast \text{expr} + \text{expr} \Rightarrow 
\]
Example Derivation

\[
\text{assign} \Rightarrow \\
\quad id := expr \Rightarrow \\
\quad A := expr \Rightarrow \\
\quad A := \text{expr} + \text{expr} \Rightarrow \\
\quad A := \text{expr} \times \text{expr} + \text{expr} \Rightarrow \\
\quad A := id \times \text{expr} + \text{expr} \Rightarrow \\
\quad A := B \times \text{expr} + \text{expr} \Rightarrow \\
\quad A := B \times id + \text{expr} \Rightarrow 
\]
Example Derivation

assign ⇒

\[ id := \text{expr} \Rightarrow \]
\[ A := \text{expr} \Rightarrow \]
\[ A := \text{expr} + \text{expr} \Rightarrow \]
\[ A := \text{expr} \times \text{expr} + \text{expr} \Rightarrow \]
\[ A := id \times \text{expr} + \text{expr} \Rightarrow \]
\[ A := B \times \text{expr} + \text{expr} \Rightarrow \]
\[ A := B \times id + \text{expr} \Rightarrow \]
\[ A := B \times A + \text{expr} \Rightarrow \]
Example Derivation

assign ⇒

 id := expr ⇒
A := expr ⇒
A := expr + expr ⇒
A := expr * expr + expr ⇒
A := id * expr + expr ⇒
A := B * expr + expr ⇒
A := B * id + expr ⇒
A := B * A + expr ⇒
A := B * A + id ⇒
Example Derivation

assign ⇒
  id := expr ⇒
A := expr ⇒
A := expr + expr ⇒
A := expr * expr + expr ⇒
A := id * expr + expr ⇒
A := B * expr + expr ⇒
A := B * id + expr ⇒
A := B * A + expr ⇒
A := B * A + id ⇒
A := B * A + C ⇒
Concrete Parse Tree
A := B * (A + C). \textit{expr} can be replaced by the kind of expression +, *. \textit{id} contains no information. Parentheses are no longer needed.
If \( A := (B \times A) + C \) was the statement, then the abstract syntax would be different and so no important information would be lost.
Ambiguity

There are two distinct parse trees for:

```
id := id + id * id
```

in the expression grammar.
Ambiguity

1. \( \text{assign} \)
   \[ \text{id} := \text{expr} \]
   \[ \text{expr} * \text{expr} \]
   \[ \text{id} \text{ expr} + \text{expr} \]
   \[ \text{id} \text{ id} \]

2. \( \text{assign} \)
   \[ \text{id} := \text{expr} \]
   \[ \text{expr} + \text{expr} \]
   \[ \text{expr} * \text{expr} \]
   \[ \text{id} \text{ id} \]
There are two distinct parse trees for:

if cond then if cond then stmt else stmt

in the typical grammar for statements in a programming language.
Ambiguity

1. stmt
   ├── if cond then stmt
   │    └── stmt
   └── stmt
e
2. stmt
   ├── if cond then stmt else stmt
   │    └── stmt
   └── stmt

if cond then stmt
Abstract Syntax Tree

Concrete parse trees may have a lot of extra stuff and may be inconvenient to use directly.
(Webber, Section 3.8 Abstract Syntax Trees, page 38.)
Example Derivation

\[
\begin{align*}
assign \Rightarrow \\
\text{id} & := \text{expr} \Rightarrow \\
A & := \text{expr} \Rightarrow \\
A & := \text{id} \ast \text{expr} \Rightarrow \\
A & := B \ast \text{expr} \Rightarrow \\
A & := B \ast (\text{expr}) \Rightarrow \\
A & := B \ast (\text{id} + \text{expr}) \Rightarrow \\
A & := B \ast (A + \text{expr}) \Rightarrow \\
A & := B \ast (A + \text{id}) \Rightarrow \\
A & := B \ast (A + C)
\end{align*}
\]
An *ambiguous grammar* is one in which some sentence has more than one parse tree. [Sebesta, 3.3.1.7 Ambiguity, page 132ff.]

The grammar of Example 3.3 [Sebesta, 7th, page 132], simple expressions, is ambiguous.

Also, this grammar of if/then/else statements is ambiguous.

Sebesta, 7th, 3.3.1.10 An Unambiguous Grammar for if–then–else, page 137; Figure 3.5, page 138.
Equivalent Grammar

Just because a grammar is ambiguous does not mean that all grammars for the same language are ambiguous.

\[ S \rightarrow \text{if } C \text{ then } S \mid \text{if } C \text{ then } S \text{ else } S \mid S' \]

This grammar for the same language is not.

\[ S \rightarrow MS \mid UMS \]
\[ MS \rightarrow \text{if } C \text{ then } MS \text{ else } MS \mid S' \]
\[ UMS \rightarrow \text{if } C \text{ then } S \mid \text{if } C \text{ then } MS \text{ else } UMS \]
If the “natural” grammar is ambiguous, could it mean that the construct is confusing to programmers? Is another approach possible? Many languages have redesigned the syntax of the if statement:

\[
S \rightarrow \begin{align*}
& \text{if } C \text{ then } S \text{ endif} \\
& \text{if } C \text{ then } S \text{ else } S \text{ endif} \\
& S'
\end{align*}
\]

The two statements with different structure now have different syntax.

\[
\begin{align*}
\text{if } C_1 \text{ then if } C_2 \text{ then } S_1 \text{ endif else } S_2 \text{ endif} \\
\text{if } C_1 \text{ then if } C_2 \text{ then } S_1 \text{ else } S_2 \text{ endif endif}
\end{align*}
\]
Perl and Go require curly braces around the then part and the else part. Python requires indenting.
Tools

BNF (LALR)

bison/yacc

token stream

parser

parse tree
The study of syntax has made the creation of the front-end of a compiler so convenient. All that is required is to specify the desired syntax and the automatical implementation is produced.
Grammars in the right form permit a parser of simple recursive functions. Each nonterminal turns into a recursive function forming a set of mutually recursive functions. Each function does a case analysis on the input to determine which production/RHS to follow. Terminals in the RHS are matched (consumed) and nonterminals turned into recursive calls.

Java Applet demo:
http://cswebsrv.cs.binghamton.edu/~zdu/parsdemo/recframe.html
http://www.uni-paderborn.de/fachbereich/AG/agkastens/compiler/parsdemo/recframe.html

See example in Figure 2.10 in Scott.
Pattern for blocks:

\[
\langle block \rangle ::= \langle block \ identifier \rangle : \\
\textbf{begin} \langle statement \rangle \{\langle statement \rangle \}
\]

Pattern for calls:

\[
\langle call \rangle ::= \langle identifier \rangle (\{\langle expression \rangle \})
\]
Beyond BNF/CFG

Formalisms with greater expressive power

▶ unrestricted grammars
▶ Post systems
▶ attribute grammars

Such formalisms are used in describing semantics — a later topic.

Notice the pragmatic definition of *syntax* (at or below context-free) and *semantics* (beyond context-free).

The set of strings

\[ \{a^n \cdot b^n \cdot c^n\} = \{\epsilon, abc, aabbc, aaabbbccc, \ldots\} \]

is not context free!
programming languages

formal languages

compilers

implementation, recognition

expressivity

language design, description, semantics