1. The are two bits: 0 and 1 (other names are: False and True, no and yes, off and on, low and high). Write down all bit strings of length:
   (a) 1
   (b) 2
   (c) 3

2. Tell me the number of bit strings of length:
   (a) 4.
   (b) 8.
   (c) \( n \), where \( n \) is a natural number, that is \( n \in \mathbb{N} = \{0, 1, 2, 3, \ldots \} \)
   (d) Does your formula (function \( n \mapsto \text{number of bit strings} \)) make sense at the boundary where \( n = 0 \)?

3. Most computers have 32-bit or 64-bit words. How many different binary words can be written using:
   (a) 32 bits?
   (b) 64 bits?

4. Do you see how to generalize your answers from above? How many different:
   (a) Decimal strings can be written using \( n \) digits?
   (b) Hexadecimal strings can be written using \( n \) hexadecimal numerals?
   (c) Lowercase English strings can be written using \( n \) letters?

5. The range of natural numbers that can be represented using 1 bit is 0 to 1. What range of natural numbers that can be represented using:
   (a) 2 bits?
   (b) 3 bits?
   (c) 4 bits?
   (d) 8 bits?
   (e) What function maps the number of bits \( n \) to the largest (top) natural number in the range?
   (f) Does your formula (function \( n \mapsto \text{top of the range} \)) make sense at the boundary where \( n = 0 \)?

6. Do you see how to generalize your answers from above? What is the range of natural numbers that can be written using:
   (a) \( n \) digits?
   (b) \( n \) hexadecimal numerals?

7. Computers have a memory address register (MAR) that holds the address of a location in memory. If the MAR is 14 bits wide, how many memory locations can be addressed?
8. Binary addition is straightforward: \(0 + 0 = 0\), \(0 + 1 = 1 + 0 = 1\) and \(1 + 1 = 0\) with a carry of 1. Compute the following binary sums:

\[
\begin{array}{c}
1 & 1 & 1 & 0 \\
+ & 0 & 1 & 1 & 0 \\
\hline
1 & 0 & 1 & 0 & 0 \\
\end{array}
\quad
\begin{array}{c}
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
+ & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
\hline
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
\end{array}
\]

9. Binary multiplication is straightforward too: \(0 \cdot x = x \cdot 0 = 0\) and \(1 \cdot x = x \cdot 1 = x\). Compute the following binary products.

\[
\begin{array}{c}
1 & 1 & 1 & 0 \\
\times & 1 & 0 & 0 \\
\hline
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
\end{array}
\quad
\begin{array}{c}
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
\times & 1 & 1 \\
\hline
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
\end{array}
\]

Total Points: 0

2015-01-12 to 2015-01-16