Name:

CSE 1400 — Applied Discrete Mathematics

Spring 2015 — Week 3 Practice

1. Let's reason about Boolean logic. Consider one of De Morgan's laws:

   $$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

   If the left-hand side is `True`, then $P \wedge Q$ is `False`. That means at least one of P or Q is `False`. And that means at least one of $\neg P$ or $\neg Q$ is `True`, and therefore, $\neg P \vee \neg Q$ is `True`.

   In the other case, if the left-hand side is `False`, then $P \wedge Q$ is `True`. That means both P and Q are `True`. And that means both of $\neg P$ and $\neg Q$ are `False`, and therefore, $\neg P \vee \neg Q$ is `False`.

   Mimic what I just wrote to argue that the second of De Morgan's laws is `True`.

   $$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

2. Here's the mechanical, truth table, proof of De Morgan's law. Rename the left and right-hand sides of De Morgan's laws $R = \neg(P \wedge Q)$ and $S = \neg P \vee \neg Q$. My argument had two steps:

   (a) $R \Rightarrow S$. If R was `True`, then S was also `True`.

   (b) $\neg R \Rightarrow \neg S$. If R was `False`, then S was also `False`.

   Fill in the truth table for $(R \Rightarrow S) \wedge (\neg R \Rightarrow \neg S)$ to conclude that it be reduced to $R \equiv S$.

R	S	$(R \Rightarrow S)$	$\wedge$	$(\neg R \Rightarrow \neg S)$

3. Last week you constructed truth tables to demonstrate that

   $$((P \wedge Q) \Rightarrow R) \equiv (P \Rightarrow (Q \Rightarrow R))$$

   This equivalence is called currying, named for Haskell Curry who explained its power in logic and computation: It reduces the computation of a function of many arguments to the composition of many functions of a single argument.

   Write an explanation of the equivalence in English. To do this, consider, in sequence, the possible values of P, Q, and R.

4. Resolution is another powerful inference tool. It says

   $$((P \Rightarrow Q) \wedge (\neg P \Rightarrow R)) \Rightarrow (Q \vee R)$$

   (a) Construct a truth table to demonstrate resolution is a valid conditional.

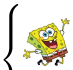
   (b) Give an English interpretation of resolution.

   (c) Look at your truth table. See that there are only two rows where $(P \Rightarrow Q) \wedge (\neg P \Rightarrow R)$ and $Q \vee R$ fail to be equivalent. Explain these two cases.

5. Set theory and Boolean logic share a common structure: They, with their operations, are both Boolean algebras. One commonality is the count of basic things.

   (a) Review your answers to question 7 from last week's homework:
   How many bit strings of length n are there?
   How many truth assignments can be made on n Boolean variables?
   A complete truth table for an n variables Boolean expression will have how many rows?

   (b) How many subsets does the set

   $\mathbb{SBSP} = \Big\{$ , ,  $\Big\}$ have?

   (c) How many subsets does an n element set have?

6. In logic you thought about passing truth assignments through a function. For instance, you learned about some 2-input Boolean functions like AND, OR, EQUIVALENT, EXCLUSIVE-OR, and IMPLIES. These functions can be visualized using a diagram such as this.

$$\begin{array}{l} \text{P} \longrightarrow \\ \text{Q} \longrightarrow \end{array} \boxed{\; b(\cdot,\,\cdot) \;} \longrightarrow b(\text{P},\,\text{Q})$$

Where $b(\cdot,\,\cdot)$ is a generic name for a 2-input Boolean function.

   How many truth assignments does a two-input Boolean function have?

   How many two-input Boolean functions are there?

   How many n-input Boolean functions are there?

   Truth tables are another way to describe Boolean functions. A complete truth table for an n variables Boolean expression has how many rows?

   How many bit strings of length 2^n are there?

   In how many ways can 2^n bit strings be map to `True` or `False`?

   In a truth table with 2^n rows, how many columns of 0's and 1's can be constructed?

7. The conditional operator $\text{P} \Rightarrow \text{Q}$ is equivalent to $\neg\text{P} \vee \text{Q}$, which can be converted mechanically into set notation $\neg\mathbb{P} \cup \mathbb{Q}$ (don't think of $\mathbb{P}$ and $\mathbb{Q}$ as the sets of prime and rational numbers: Here they just name matching the logic notation)

   Fill in the truth table

$x \in \mathbb{P}$	$x \in \mathbb{Q}$	$x \in \neg\mathbb{P}$	$\vee$	$x \in \mathbb{Q}$

   Make a conjecture about how to interpret the Boolean operator $\Rightarrow$ in the context of sets.

8. Set theory and Boolean logic share a common structure: But, notationally set theory uses round symbols while pointy symbols are used in logic.

Match the natural equivalences between set theory and Boolean algebra notation. What are the common names for each symbol? Be certain you understand the meanings of the names and their context.

Logic symbols

(a) $\wedge$
(b) $\{$`True`, `False`$\}$
(c) $\vee$
(d) $\Rightarrow$

Set symbols

(a) $\in$
(b) $\cup$
(c) $\cap$
(d) $\subseteq$

9. Let $\mathbb{EVEN} = \{0, 2, 4, 6, 8\}$, $\mathbb{ODD} = \{1, 3, 5, 7, 9\}$ and $\mathbb{PRIME} = \{2, 3, 5, 7\}$ be the *even*, *odd* and *prime* digits. Compute the following set operations over the universe of digits $\mathbb{U} = \mathbb{D} = \{0, 1, 2, \ldots, 8, 9\}$.

   (a) $\neg\mathbb{EVEN}$
   (b) $\neg\mathbb{PRIME}$
   (c) $\mathbb{ODD} \cup \mathbb{PRIME}$
   (d) $\mathbb{EVEN} \cup \mathbb{PRIME}$
   (e) $\mathbb{ODD} \cap \mathbb{PRIME}$
   (f) $\mathbb{EVEN} \cap \mathbb{PRIME}$
   (g) $\mathbb{EVEN} \cap \mathbb{ODD}$
   (h) $\mathbb{ODD} \cap \neg\mathbb{PRIME}$

10. Reasoning about nothing can be difficult. Reasoning about everything is difficult too. Which of the following statements about the empty set are `True` and which are `False`? Explain your answers. Recall $\in$ means "is an element of" and $\subseteq$ means "is a subset of."

   (a) $\emptyset = \{\emptyset\}$
   (b) $\emptyset \in \{\emptyset\}$
   (c) $\emptyset \in \emptyset$
   (d) $\{\emptyset\} \in \emptyset$
   (e) $\{\emptyset\} \in \{\emptyset\}$
   (f) $\emptyset \subseteq \{\emptyset\}$
   (g) $\emptyset \subseteq \emptyset$
   (h) $\{\emptyset\} \subseteq \emptyset$
   (i) $\{\emptyset\} \subseteq \{\emptyset\}$

Total Points: 0 2015-01-26 to 2015-01-30