Name:

CSE 1400 Applied Discrete Mathematics

Spring 2015 Week 5 Practice

1. Functions are a crucial concept: A function is a way to assign values from a domain to values in a co-domain. Functions obey a rule:

   A value in the domain can map to a only one value in its co-domain.

   In this class we, almost always, deal with *total* functions. That is, *each* value in the domain is mapped to some value in the range. (Partial functions, where the function is not defined on some values in the domain, are interesting and important, but let's stick to the basics.)

   In your previous study of mathematics, you learned to draw graphs of some simple functions. Draw the graphs of these functions on the indicated intervals.

   (a) $y = x^2 - x - 1$, the Fibonacci parabola for $-2 \leq x \leq 3$.
   (b) $y = \lg x$, the logarithm base 2 for $0.125 \leq x \leq 8$.
   (c) $y = 2^x$, the exponential base 2 (the inverse of $\lg x$) for $-3 \leq x \leq 3$.
   (d) $y = \cos x$, the cosine function for $0 \leq x \leq 4\pi$. (Although little used in elementary discrete mathematics, the cosine is related to inner products in Hilbert spaces, and then things really become interesting.)

2. To understand the idea of *function* you much also understand the idea of of Cartesian products. Let $\mathbb{X}$ and $\mathbb{Y}$ be sets. The Cartesian product $\mathbb{X} \times \mathbb{Y}$ is the set

   $$\mathbb{X} \times \mathbb{Y} = \{(x, y) : x \in \mathbb{X} \quad \text{and} \quad y \in \mathbb{Y}\}$$

   Cartesian coordinates are introduced as the xy-plane in a beginning algebra class.

   A function is a mapping from $\mathbb{X}$ to $\mathbb{Y}$, denoted

   $$f : \mathbb{X} \mapsto \mathbb{Y}$$

   such that if $f(x) = y_1$ and $f(x) = y_2$, then $y_1 = y_2$. A function a subset of $\mathbb{X} \times \mathbb{Y}$ such that the first coordinate x occurs once and only once ordered pairs (x, y) belonging to the function.

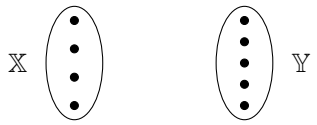
   (a) What is the cardinality of $\mathbb{D} \times \mathbb{H}$?
   (b) Since a set $\mathbb{X}$ with cardinality n has 2^n subsets, how many subsets does $\mathbb{D} \times \mathbb{H}$ have?
   (c) How many points are there in a (total) function $f : \mathbb{D} \mapsto \mathbb{H}$?
   (d) Many different (total) function $f : \mathbb{D} \mapsto \mathbb{H}$ are there?
   (e) How many points are there in a (total) function $f : \mathbb{H} \mapsto \mathbb{D}$?
   (f) Many different (total) function $f : \mathbb{H} \mapsto \mathbb{D}$ are there?

3. Onto is a basic property some functions have. Using predicate logic to write the definition of an onto function.

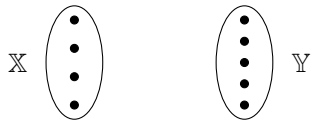
4. One-to-one is a basic property some functions have. Using predicate logic to write the definition of an one-to-one function.

5. By drawing arrows from values in $\mathbb{X}$ to values in $\mathbb{Y}$ show how to construct examples of the following types, or explain why no such example can be drawn. (Remember: functions are total, an arrow must leave each value in $\mathbb{X}$.)

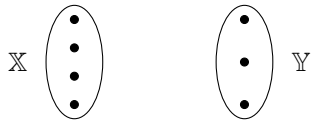
(a) Draw a graph that is not a function.



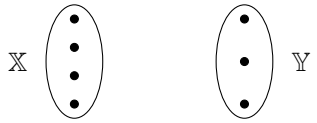
(b) Draw a graph of a function that is onto.



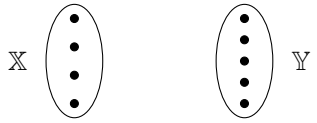
(c) Draw a graph of a function that is onto.



(d) Draw a graph of a function that is one-to-one.



(e) Draw a graph of a function that is one-to-one.



6. Stuffing pigeons into pigeonholes is a crucial idea, but it may not be good for the health of pigeons.

   (a) Why does 'more pigeons than pigeonhole' violate one-to-one?

   (b) Why does 'fewer pigeons than pigeonhole' violate onto?

   (c) Pretend you stuff 237 pigeons into 13 pigeonholes. Why must there be a pigeonhole with 18 or fewer pigeons?

   (d) Pretend you stuff 237 pigeons into 13 pigeonholes. Why must there be a pigeonhole with 19 or more pigeons?

7. In the movie "Cheaper by the Dozen," there are 12 children and 2 parents in the family.

   (a) Prove that at least two of the children were born on the same day of the week.

   (b) Prove that at least two family members (including mother and father) are born in the same month.

   (c) Assuming there are 4 children's bedrooms in the house, show that there are at least 3 children sleeping in at least one of them.

Total Points: 0 2015-02-09 to 2015-02-13