Factorials

The factorial \( n! \) is the product of integers from \( n \) down to 1.

- \( 1! = 1 \) (one factorial is 1)
- \( 2! = 2 \cdot 1 = 2 \) (two factorial is 2 times 1)
- \( 3! = 3 \cdot 2 \cdot 1 = 6 \) (three factorial is 3 times 2 times 1)
- \( 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24 \) (four factorial is 4 times 3 times 2 times 1)
- In general, \( n! = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1 \)

Zero Factorial

Notice that \( n! \) is the product of \( n \) factors. For instance, \( 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \).

If that is the case, then \( 0! \) is the product of no factors.

The product of no factors is called the empty product.

Since 1 is the multiplicative identity, that is, \( a \cdot 1 = a \), it makes sense to define the value of the empty product to be 1.

Therefore, \( 0! = 1 \) (zero factorial is one).

Factorial Form of the Binomial Coefficient

If you can accept for the moment that

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]

And, if you can compute

\( n! = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1 \)

Then you can compute binomial coefficients.
Factorial Form of the Binomial Coefficient

Consider the following instances of the binomial coefficient:

- \( \binom{n}{0} \) is
  \[
  \binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \cdot n!} = 1
  \]

- \( \binom{7}{3} \) is
  \[
  \binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \cdot 4!} = \frac{7 \cdot 6 \cdot 5}{3!} = 7 \cdot 5 = 35
  \]

- \( \binom{52}{5} \) is
  \[
  \binom{52}{5} = \frac{52!}{5! \cdot 47!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = 2,598,960
  \]