The Gauss Sequence

The Gauss sequence is my name for the sequence of natural numbers.

\[ \vec{G} = \langle 0, 1, 2, 3, 4, 5, 6, 7, \ldots \rangle \]

When Gauss was only 10, he discovered a simple function to compute the sum of the first \( n \) natural numbers.

\[
0 + 1 + 2 + 3 + 4 + \cdots + (n-2) + (n-1) = \frac{n(n-1)}{2}
\]

The Gauss Sequence

The Gauss sequence is arithmetic.

Terms in the Gauss sequence lie on the 45° line \( y = x \).
The Gauss Sequence in Pascal’s Triangle

The Gauss sequence is column 1 of Pascal’s rectangle.

| Binomial Coefficients \( \binom{n}{k} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( n \)       | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0           | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1           | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2           | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3           | 1 | 3 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4           | 1 | 4 | 6 | 4 | 1 | 0 | 0 | 0 | 0 | 0 |
| 5           | 1 | 5 | 10 | 10 | 5 | 1 | 0 | 0 | 0 | 0 |
| 6           | 1 | 6 | 15 | 20 | 15 | 6 | 1 | 0 | 0 | 0 |
| 7           | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | 0 | 0 |
| 8           | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | 0 |
| 9           | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 |

The Gauss Sequence is the Sum of Alice Terms

Summing terms in the Alice sequence produces the Gauss sequence.

- The sum of no Alice terms (the empty sum) is 0, term 0 in the Gauss sequence.
- The sum of one Alice term, \( 1 + 1 = 2 \), term 2 in the Gauss sequence.
- The sum of two Alice terms, \( 1 + 1 + 1 = 3 \), term 3 in the Gauss sequence.
- The sum of three Alice terms, \( 1 + 1 + 1 + 1 = 4 \), term 4 in the Gauss sequence.

Computing Terms in the Gauss Sequence

Terms in the Gauss sequence can be computed by the function

\[
g(n) = n \quad \text{for all natural numbers } n
\]

Gauss terms can also be computed by an initial condition

\[
g_0 = 0 \quad \text{(the first term is 0)}
\]

and a recurrence equation

\[
g_n = g_{n-1} + 1
\]

(the next term equals the previous term plus 1), \( \forall n \in \mathbb{N}, n \geq 1 \)