Horner’s Rule

Horner’s rule is an efficient algorithm for converting a number written in base $b$ into its decimal notation. Horner’s rule is also useful for evaluating a polynomial, and Taylor coefficients.

Evaluating polynomials by Horner’s rule is covered elsewhere in this course.

Consider the natural number 43. Writing 43 as a sum of powers of 2 implies its binary representation.

$$43 = 32 + 8 + 2 + 1$$

$$43 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$43 = (101011)_2$$

Horner’s rule writes converts $(101011)_2$ to 43 from the most significant bit to the least significant bit.

$$43 = (((1 \cdot 2 + 0) \cdot 2 + 1) \cdot 2 + 0) \cdot 2 + 1$$

This multiplicity of parentheses is somewhat confusing, so let’s re-formulate it.

Horner’s Rule to Convert Binary to Decimal

To convert $(101011)_2$ to decimal:

1. Write the bit pattern to be converted (leave sufficient space between bits)

```
1 0 1 0 1 1
```

2. Next, bring down the leading 1

```
1 0 1 0 1 1
```

(Continued next slide)
Example Continued

Continuing from the previous slide.

3. Multiply the 1 by 2 and place it under the next bit.

\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 1 \\
2 & & & & & \\
\hline
1 & & & & &
\end{array}
\]

4. Add the values in the second column.

\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 1 \\
2 & & & & & \\
\hline
1 & 2 & & & &
\end{array}
\]

5. Repeat the process.

Example Continued

Continuing from the previous slides.

Multiply 2 by 2 and add 1.

\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 1 \\
2 & 4 & & & & \\
\hline
1 & 2 & 5 & & &
\end{array}
\]

Multiply 5 by 2 and add 0.

\[
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 1 \\
2 & 4 & 10 & & & \\
\hline
1 & 2 & 5 & 10 & &
\end{array}
\]
Example Continued

Continuing from the previous slides.

Multiply 10 by 2 and add to 1.

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 1 & 1 \\
2 & 4 & 10 & 20 \\
\hline
1 & 2 & 5 & 10 & 21
\end{array}
\]

Multiply 21 by 2 and add 1.

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 1 & 1 \\
2 & 4 & 10 & 20 & 42 \\
\hline
1 & 2 & 5 & 10 & 21 & +3
\end{array}
\]

Therefore, \((10 1011)_2 = 43\).

Other Examples of Binary to Decimal Conversion

Here’s an example computation of Horner’s rule to convert binary \((1101 0100)_2\) to decimal 212.

\[
\begin{array}{cccccccc}
\text{Horner’s Rule} \\
1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
2 & 6 & 12 & 26 & 52 & 106 & 212 \\
\hline
1 & 3 & 6 & 13 & 26 & 53 & 106 & 212
\end{array}
\]

And here is a second example.

\[
\begin{array}{cccccccc}
\text{Horner’s Rule} \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
2 & 4 & 8 & 16 & 34 & 70 & 142 \\
\hline
1 & 2 & 4 & 8 & 17 & 35 & 71 & 143
\end{array}
\]

Therefore, \((1000 1111)_2 = 143\).
Converting Ternary to Decimal

You can also use Homer's rule to convert from bases other than 2 to decimal.

Here's an example computation of Homer's rule to convert ternary \((210)_3\) to decimal 21.

\[
\begin{array}{c|ccc}
\text{Homer's Rule base}=3 \\
2 & 1 & 0 \\
\hline
6 & 21 \\
\hline
2 & 7 & 21
\end{array}
\]

Converting Hexadecimal to Decimal

Here's an example computation of Homer's rule to convert hexadecimal \((CAFE)_{16}\) to decimal 51966.

\[
\begin{array}{c|ccc}
\text{Homer's Rule base}=16 \\
C & A & F & E \\
\hline
192 & 3232 & 51952 \\
\hline
12 & 202 & 3247 & 51966
\end{array}
\]

Because \((DEAD\ CODE)_{16}\) is an interesting string, you might want to convert it to decimal.