Horner's Rule

Horner's rule is an efficient algorithm for converting a number written in base b into its decimal notation.

Horner's rule is also useful for evaluating a polynomial, and Taylor coefficients.

Evaluating polynomials by Horner's rule is covered elsewhere in this course.

Horner's Rule

Consider the natural number 43. Writing 43 as a sum of powers of 2 implies its binary representation.

$$43 = 32 + 8 + 2 + 1$$

$$43 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$43 = (10\ 1011)_2$$

Horner's rule writes converts $(10\ 1011)_2$ to 43 from the most significant bit to the least significant bit.

$$43 = (((((1 \cdot 2) + 0) \cdot 2 + 1) \cdot 2 + 0) \cdot 2 + 1)$$

This multiplicity of parentheses is somewhat confusing, so let's reformulate it.

Horner's Rule to Convert Binary to Decimal

To convert $(10\ 1011)_2$ to decimal:

1. write the bit pattern to be converted (leave sufficient space between bits)

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ \hline & & & & & \square \end{array}$$

2. Next, bring down the leading 1

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ \hline 1 & & & & & \square \end{array}$$

(Continued next slide)

Example Continued

Continuing from the previous slide.

3. Multiply the 1 by 2 and place it under the next bit.

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ & 2 & & & & \\ \hline 1 & & & & & \square \end{array}$$

4. Add the values in the second column.

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ & 2 & & & & \\ \hline 1 & 2 & & & & \square \end{array}$$

5. Repeat the process.

Example Continued

Continuing from the previous slides.

Multiply 2 by 2 and add 1.

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ & 2 & 4 & & & \\ \hline 1 & 2 & 5 & & & \square \end{array}$$

Multiply 5 by 2 and add 0.

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ & 2 & 4 & 10 & & \\ \hline 1 & 2 & 5 & 10 & & \square \end{array}$$

Example Continued

Continuing from the previous slides.

Multiply 10 by 2 and add to 1.

1	0	1	0	1	1
	2	4	10	20	
1	2	5	10	21	$\square$

Multiply 21 by 2 and add 1.

1	0	1	0	1	1
	2	4	10	20	42
1	2	5	10	21	$\boxed{43}$

Therefore, $(10\ 1011)_2 = 43$.

Other Examples of Binary to Decimal Conversion

Here's an example computation of Horner's rule to convert binary $(1101\ 0100)_2$ to decimal 212.

Horner's Rule							
1	1	0	1	0	1	0	0
	2	6	12	26	52	106	212
1	3	6	13	26	53	106	$\boxed{212}$

And here is a second example.

Horner's Rule							
1	0	0	0	1	1	1	1
	2	4	8	16	34	70	142
1	2	4	8	17	35	71	$\boxed{143}$

Therefore, $(1000\ 1111)_2 = 143$.

Converting Ternary to Decimal

You can also use Horner's rule convert from bases other than 2 to decimal.

Here's an example computation of Horner's rule to convert ternary $(210)_3$ to decimal 21.

Horner's Rule Base=3		
2	1	0
	6	21
2	7	$\boxed{21}$

Converting Hexadecimal to Decimal

Here's an example computation of Horner's rule to convert hexadecimal $(CAFE)_{16}$ to decimal 51966.

Horner's Rule Base=16			
C	A	F	E
	192	3232	51952
12	202	3247	$\boxed{51966}$

Because $(DEAD\ CODE)_{16}$ is an interesting string you might want to convert it to decimal.