Sets

A set is a well-defined collection of distinct objects.

Sets are denoted by capital letters, written in blackboard bold font.

For instance,

$$\mathbb{A}, \mathbb{B}, \mathbb{C}, \ldots, \mathbb{X}, \mathbb{Y}, \mathbb{Z}$$

The objects in a set are called <u>elements</u> or <u>members</u>.

That object x is an element of set $\mathbb{X}$ is denote by the notation

$$x \in \mathbb{X}$$

Important Finite Sets

There are several sets that are important in discrete mathematics.

- The set of bits $\mathbb{B} = \{0, 1\}$.
- The set of digits $\mathbb{D} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
- The set of hexadecimal numerals

$$\mathbb{H} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

All of the above are finite sets.

Important Countably Infinite Sets

There are several countable sets that are important in discrete mathematics.

- The set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$.
- The set of integers $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \ldots\}$.
- The set of rational numbers $\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z} \text{ and } b \neq 0\right\}$.

All of the above are countable sets.

Important Uncountable Sets

There are several uncountable sets that are important in continuous mathematics.

- The set of real numbers $\mathbb{R} = \{x : x \text{ is the limit of a Cauchy sequence}\}$. (Not that you are expected to understand what this means.)
- The set of complex numbers $\mathbb{C} = \{x + iy : x, y \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$.

The study of these sets is beyond the scope of discrete mathematics and this course.

The Empty Set

There is one set that does not have any elements.

This set is called the empty set.

There are two commonly used notations for the empty set.

1. I use the symbol $\emptyset$, a Scandinavian letter.
2. Because it is easy to typeset, some people use a pair of empty curly braces $\{\}$.

The statement $x \in \emptyset$ is `False`.

The statement $x \notin \emptyset$ is `True`.

Universes

To model a theory requires context.

A universe or universal set $\mathbb{U}$ helps to provide this context.

The universe $\mathbb{U}$ is the set of all elements under consideration.

Depending on context, several universes we will occur in this course: Bits, Digits, Natural numbers, etc.

The statement $x \in \mathbb{U}$ is `True`.

The statement $x \notin \mathbb{U}$ is `False`.

Sets of Sets

Is it common to have a universe $\mathbb{U}$ that is a collection of sets.

For instance,

$$\mathbb{U} = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$$

Later, you will learn that the set of all subsets of a set together with the set operations: Complement, Intersection, and Union have the same algebraic structure as Boolean logic.