The Tower of Hanoi

After God formed the Earth, God established an order of monks to move 64 golden disks from one diamond needle onto another using a third diamond needle for temporary storage.

The monks must move the disks subject to God's laws.

1. Move only one disk at a time.

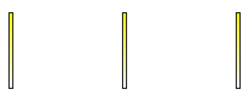
2. Never place a large disk on a smaller one.

When the monks complete their task, the world will end.

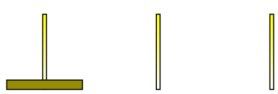
The Tower of Hanoi

To compute when the world will end, let's consider some small cases, then generalize the result and compute our demise.

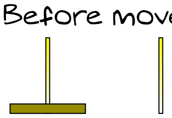
The Tower of Hanoi with No Disks and 1 Disk



No disks: No moves

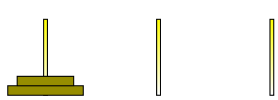


One disk: Before move

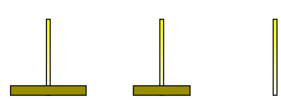


One disk: After one move

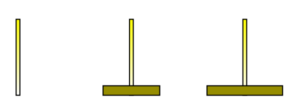
The Tower of Hanoi with 2 Disks



Two disks: Before moves



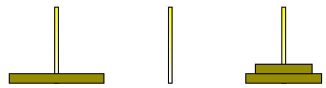
Two disks: After one move



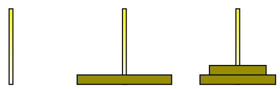
Two disks: After two moves

Two dis

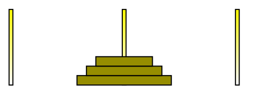
The Tower of Hanoi with 3 Disks



Three disks: After three moves



Three disks: After four moves



Three disks: After seven moves

Tower of Hanoi: Disks Versus Moves

Notice the pattern of disks versus moves.

Number of Disks	$n =$	0	1	2	3	4	5	6
Number of Moves	$m_n =$	0	1	3	7	15	31	63

This is the Mersenne sequence.

To move n disks, move the top $n-1$ disks, move the bottom (largest) disk, then move the top $n-1$ disks back onto the bottom disk.

$$m_n = m_{n-1} + 1 + m_{n-1} = 2m_{n-1} + 1$$

The function that computes m_n is

$$m(n) = 2^n - 1$$

When Will the World End?

To move n disks requires

$$m(n) = 2^n - 1 \quad \text{moves}$$

To move 64 disks requires

$$m(64) = 2^{64} - 1 \quad \text{moves}$$

How long is this?
Let's assume monks can move 1 disk every second.

When Will the World End?

Let's do a back-of-the-envelope calculation.

As you know, there are about π billion seconds in a century.

$$\begin{aligned}
1 \text{ century} &= 100 \text{ years} \\
&\approx 36,500 \text{ days} \\
&= 36,500 \times 24 = 876,000 \text{ hours} \\
&= 876,000 \times 60 = 52,560,000 \text{ minutes} \\
&= 52,560,000 \times 60 = 3,153,600,000 \text{ seconds} \\
&\approx \pi \text{ billion seconds}
\end{aligned}$$

When Will the World End?

Here's another useful approximation.

$$2^{10} = 1,024 \approx 1,000 = 10^3$$

You can use this to approximate the number of moves for 64 disks.

When Will the World End?

The monks can complete their task in

$$2^{64} - 1 \text{ moves} = \left(2^{10}\right)^{6.4} - 1 \text{ seconds}$$

This is about $\left(10^3\right)^{6.4} = 10^{19.4}$ seconds. Or, approximating again, 10^{10} billion seconds. And since $3\pi \approx 10$, the monks can complete their task in

$$\approx 3 \text{ billion centuries}$$

Scientist estimate the age of the universe to be 13.75 billion years, or about 137 million centuries.

That is, the monks will finish when the universe is about 20 time its current age!