In Class Problem Set #17

CSE 1400 and MTH 2051

Fall 2012

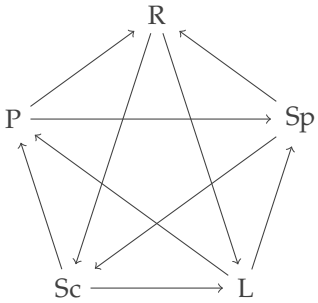
Representing Relations

1. Consider the rules of "Rock-Paper-Scissors-Lizard-Spock"

- Scissors cuts paper
- Paper covers rock
- Rock crushes lizard
- Lizard poisons Spock
- Spock smashes scissors
- Scissors decapitates lizard
- Lizard eats paper
- Paper disproves Spock
- Spock vaporizes rock
- Rock crushes scissors

(a) Draw a graph which shows the rules of "Rock-Paper-Scissors-Lizard-Spock"

Answer: Let R, P, Sc, L, Sp stand for Rock, Paper, Scissors, Lizard, Spock. The graph of rules is



(b) Construct an adjacency matrix which shows the rules of "Rock-Paper-Scissors-Lizard-Spock"

Answer: Let R, P, Sc, L, Sp stand for Rock, Paper, Scissors, Lizard, Spock. The adjacency matrix is

	R	P	Sc	L	Sp
R	1	0	1	1	0
P	1	1	0	0	1
Sc	0	1	1	1	0
L	0	1	0	1	1
Sp	1	0	1	0	1

(c) Is the game a partial order? Why or why not?

Answer: The answer is no, the game is not a partial order. I've made the game reflexive by my choice. By looking at the adjacency matrix, you can determine that the game is antisymmetric. However, the game is not transitive, which you can see by looking at the graph. For example, Rock smashes Lizara and Lizard poisons Spock, but Rock does not beat Spock: The edges are directed.

(d) Is the game an equivalence? Why or why not?

Answer: The answer is no, the game is not an equivalence relation. By looking at the adjacency matrix, you can determine that the game is not symmetric and the game is not transitive.

2. Construct an adjacency matrix for the divides relation on the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Answer:

	0	1	2	3	4	5	6	7	8	9
0	1	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1
2	1	0	1	0	1	0	1	0	1	0
3	0	0	0	1	0	0	1	0	0	1
4	0	0	0	0	1	0	0	0	1	0
5	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1

3. Construct an adjacency matrix for the congruence mod 5 relation on the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Answer:

	0	1	2	3	4	5	6	7	8	9
0	1	0	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
2	0	0	1	0	0	0	0	1	0	0
3	0	0	0	1	0	0	0	0	1	0
4	0	0	0	0	1	0	0	0	0	1
5	1	0	0	0	0	1	0	0	0	0
6	0	1	0	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0	1	0	0
8	0	0	0	1	0	0	0	0	1	0
9	0	0	0	0	1	0	0	0	0	1