In Class Problem Set #21

CSE 1400 and MTH 2051

Fall 2012

Naming Systems

1. How many numbers can be named using:

   (a) 1 digit?
   Answer: 10 numbers can be named: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

   (b) 2 digits?
   Answer: 10^2 numbers can be named.

   (c) 3 digits?
   Answer: 10^3 numbers can be named.

   (d) 4 digits?
   Answer: 10^4 numbers can be named.

   (e) n digits
   Answer: 10^n numbers can be named.

2. How many numbers can be named using:

   (a) 1 bit?
   Answer: 2 numbers can be named: 0, 1.

   (b) 2 bits?
   Answer: $2^2 = 4$ numbers can be named: 00, 01, 10, 11, which stand for 0 through 3.

   (c) 3 bits?
   Answer: $2^3 = 8$ numbers can be named:

   000, 001, 010, 011, 100, 101, 110, 111

   which stand for 0 through 7.

   (d) 8 bits (a byte)?
   Answer: $2^8 = 256$ numbers can be named: 0 to 255.

   (e) n bits
   Answer: 2^n numbers can be named.

3. Internet Protocol version 4 and 6 (IPv4 and IPv6) use 4 and 16 bytes to encode addresses of devices on the Internet. A rough estimate of the current world population is 7 billion people. Say a billion is roughly $2^{30} = 1,073,741,824$. Estimate the number of devices that can be assigned per person.
   Answer: Each byte can code $2^8 = 256$ values. 4 bytes can code $256^4 = 2^{32}$ addresses and 16 bytes can code $256^{16} = 2^{128}$ addresses. Therefore,

   - There are about
   $$\frac{2^{32}}{7^{30}} \approx \frac{2^{32}}{8^{30}} = \frac{1}{2}$$
   devices per person using IPv4
   - There are about
   $$\frac{2^{128}}{7^{30}} \approx \frac{2^{128}}{8^{30}} = 2^{95} \approx 4 \times 10^{28}$$
   devices per person using IPv6

4. The DNA alphabet is the set
   $$\mathbb{DNA} = \{A, C, G, T\}$$
   where A, C, G, and T are nucleotides called "bases."

(a) A sequence of 3 nucleotides code an amino acid. How many 3 base strings are there?
Answer: There are $4^3 = 64$ strings.

(b) How many DNA strings of length n are there?
Answer: There are 4^n strings.

(c) How many DNA strings are there of length 0 or 1 or 2 or $\ldots n-1$?
Answer: There are

$$\sum_{k=0}^{n-1} 4^k = \frac{4^n - 1}{3}$$

DNA strings with these lengths.