In Class Problem Set #26
CSE 1400 and MTH 2051
Fall 2012

Greatest Common Divisor

1. What is the greatest common divisor of the following pairs of natural numbers?
   (a) \(a = 2^43^55^2\) and \(b = 2^33^75^4\).
      Answer: The greatest common divisor is \(2^33^55^2\).
   (b) \(a = 7^311^5\) and \(b = 3^87^211^213^3\).
      Answer: The greatest common divisor is \(7^311^2\).

2. What is the least common multiple\(^1\) of the following pairs of natural numbers?
   (a) \(a = 2^43^55^2\) and \(b = 2^33^75^4\).
      Answer: The least common multiple is \(2^43^75^4\).
   (b) \(a = 7^311^5\) and \(b = 3^87^211^213^3\).
      Answer: The least common multiple is \(3^37^211^513^3\).

3. From the previous two problems what would you guess the product \(\gcd(a, b) \text{lcm}(a, b)\) is equal to?
   Answer: The product \(\gcd(a, b) \text{lcm}(a, b)\) is equal to \(ab\).

4. Use the Euclidean algorithm to compute the greatest common divisor of the following pairs of natural numbers.
   (a) \(\gcd(7, 10)\).

\(^1\) Students of arithmetic will know this as the least common denominator, which is used when adding fractions.
The greatest common denominator is 1.

(b) gcd(21, 49).

Answer:

\[
\begin{align*}
49 & = 21 \cdot 2 + 7 \\
21 & = 7 \cdot 3 + 0
\end{align*}
\]

The greatest common denominator is 7.

(c) gcd(117, 176).

Answer:

\[
\begin{align*}
176 & = 117 \cdot 1 + 59 \\
117 & = 59 \cdot 2 + 58 \\
59 & = 58 \cdot 1 + 1 \\
58 & = 1 \cdot 58 + 0
\end{align*}
\]

The greatest common denominator is 1.

(d) gcd(462, 1071).

Answer:

\[
\begin{align*}
1071 & = 462 \cdot 2 + 147 \\
462 & = 147 \cdot 3 + 21 \\
147 & = 21 \cdot 7 + 0
\end{align*}
\]

The greatest common denominator is 21.