In Class Problem Set #4

CSE 1400 and MTH 2051

Fall 2012

Instructions

1. Join the other students who share your number.
2. Introduce yourselves to each other.
3. As a group, complete as many of the problems as you can and record your answers.
4. Be certain each member of your group understands the answer and that you all agree it is correct.
5. If you cannot solve some problem, mark it for later thought and move to the next problem.
6. Complete all problems (with your group, by yourself, or with others) before the next class.

Predicate Logic

1. Let $L(x, y)$ represent the predicate "x loves y." Write the following statements using the quantification symbols $\forall$ (for all) and $\exists$ (there exists) and the predicate $L(x, y)$.

   (a) Everyone loves someone.
   Answer: $(\forall x)(\exists y)(L(x, y))$

   (b) Everyone is loved by someone.
   Answer: $(\exists x)(\forall y)(L(x, y))$

   (c) Everyone loves himself.
   Answer: $(\forall x)(L(x, x))$

   (d) Everyone loves everyone.
   Answer: $(\forall x)(\forall y)(L(x, x))$

   (e) Someone loves someone.
   Answer: $(\exists x)(\exists y)(L(x, y))$

   (f) Someone loves everyone.
   Answer: $(\exists x)(\forall y)(L(x, y))$

   (g) Someone loves himself.
   Answer: $(\exists x)(L(x, x))$

   (h) Someone is loved by everyone.
   Answer: $(\forall x)(\exists y)(L(x, x))$

2. Write the following English sentences using the quantification symbols $\forall$ and $\exists$ and symbols from Boolean logic ($\neg$, $\wedge$, $\vee$, $\rightarrow$, $\equiv$, $\oplus$), mathematical notation.

(a) For every prime number there is another prime number larger than the first.

Answer:

$$(\forall p \in \mathbb{P})(\exists q \in \mathbb{P})(q > p)$$

(b) Every integer is either even or odd.

Answer:

$$(\forall n \in \mathbb{Z})(\exists k \in \mathbb{Z})((n = 2k) \oplus (n = 2k+1))$$

You could use OR ($\vee$) here.

(c) For every real number, if its absolute value is greater than 1, then it is either less than -1 or greater than 1.

Answer:

$$(\forall x \in \mathbb{R})((|n| > 1) \rightarrow (n < -1) \vee (n > 1))$$

You could use an Exclusive OR ($\oplus$) here.

3. Write the following mathematical statements as English sentences.

(a) $(\forall n)(\exists k)((n = 2k) \vee (n = 2k+1))$ (The values of n and k are integers.)

Answer: Every integer is either even or odd.

(b) $(\forall n)(\exists k)((n = 4k) \vee (n = 4n+1) \vee (n = 4k+2) \vee (n = 4k+3))$ (The values of n and k are integers.)

Answer: Every integer has a remainder of 0, 1, 2, or 3 when divided by 4. Equivalently, every integer is either a multiple of 4, a multiple of 4 plus 1, a multiple of 4 plus 2, or a multiple of 4 plus 3.

(c) $(\forall p)(\exists q)(q = p+2)$ (The values of p and q lie in the set of prime numbers.)

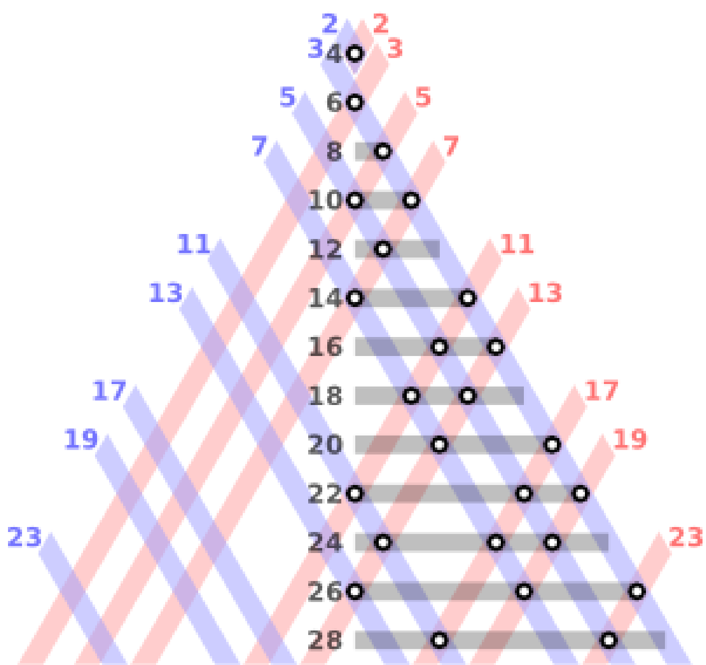
Answer: For every prime, there exists a prime q that is 2 more than p. This statement is `False`. For $p = 2$, $p+2 = 4$ is not prime. Another counterexample is $p = 7$, $p+2 = 9$ is not prime. A related statement is the twin prime conjecture: There are infinitely many twin primes, that is numbers such that p and $p+2$ are prime. It is not known if the twin prime conjecture is `True` or `False`, but it must be one of these. One way to write the twin prime conjecture is

$$(\forall n)(\exists p \in \mathbb{P})(p+2 \in \mathbb{P})$$

(d) $(\forall n > 2)(\exists p, q)(n = p + q)$ (The value of n is a natural number and the values of p and q are prime numbers.)

Answer: Every natural number greater than 2 can be written as the sum of two primes. It is `False` a counterexample is 3, 3 cannot be written as the sum of two primes. A non-trivial counterexample is 27; 27 is not the sum of two primes. If you change the statement to every even integer greater than 2, you get Goldbach's conjecture. It is not known if Goldbach's conjecture is `True` or `False`, but it must be one of these. One way to write Goldbach's conjecture is

$$(\forall n > 2)((n \bmod 2 = 0) \rightarrow (\exists p, q)(n = p + q)$$



(e) $(\forall\, x)(\exists!\, y)(f(x) = y)$ (The values of x and y are real numbers. The variable f is not bound.)

Answer: f is a function from the real numbers to the real numbers.