**Name:**

# CSE 1400 Applied Discrete Mathematics

# Fall 2012 Quiz Two

**Score**

1. (15 pts) Let $\mathbb{M}$ and $\mathbb{L}$ denote the sets of mathematicians and logicians, respectively, and let $\textsc{loves}(m, l)$ denote the predicate "$m$ loves $l$." Write the two statements below as English sentences.

   (a) $(\forall\ m \in \mathbb{M})(\exists\ l \in \mathbb{L})(\textsc{loves}(m, l))$
   Answer: Every mathematician loves some logician.

   (b) $(\exists\ l \in \mathbb{L})(\forall\ m \in \mathbb{M})(\textsc{loves}(m, l))$
   Answer: There is some logician loved by every mathematician.

   (c) Are the statements equivalent? Why or why not?
   Answer: The statements are not equivalent. In problem 1a the mathematicians don't have to love the same logician; in problem 1b they do.

**Score**

2. (5 pts) Write the English sentence below as a mathematical sentence using (as needed) Boolean logic, quantifiers, arithmetic, etc..

   "For every pair of natural numbers $a$ and $n$, there exist natural numbers $q$ and $r$ such that $a = qn + r$."

   Answer: Conceptually: When $a$ is divided by $n$, there is a quotient $q$ and a remainder $r$. (For the special case when $n = 0$, any natural number $q$ can be taken for the quotient and the remainder is $r = a$.)

$$(\forall\ a,\ n \in \mathbb{N})(\exists\ q,\ r, \in \mathbb{N})(a = qn + r)$$

**Score**

3. (25 pts) Consider universal set of 19 "word letters *".

$$\mathbb{W} = \{a,\ b,\ c,\ d,\ g,\ i,\ j,\ k,\ l,\ m,\ o,\ p,\ q,\ r,\ t,\ u,\ v,\ x,\ y\}$$

   (a) Let $\mathbb{X} = \{a,\ c,\ g,\ j,\ l,\ o,\ q,\ t,\ v,\ y\}$ and $\mathbb{Y} = \{a,\ b,\ c,\ v,\ x,\ y\}$ be subsets of $\mathbb{W}$.

      i. What is the complement of $\mathbb{X}$ with respect to $\mathbb{W}$?
      Answer: $\overline{\mathbb{X}} = \{b,\ d,\ i,\ k,\ m,\ p,\ r,\ u,\ x\}$

      ii. What is $\mathbb{X} \cup \mathbb{Y}$?
      Answer: $\mathbb{X} \cup \mathbb{Y} = \{a,\ b,\ c,\ g,\ j,\ l,\ o,\ t,\ v,\ x,\ y\}$

      iii. What is $\mathbb{X} \cap \mathbb{Y}$?
      Answer: $\mathbb{X} \cap \mathbb{Y} = \{a,\ c,\ \ v,\ y\}$

   (b) How many subsets (in total) does $\mathbb{W}$ have?
   Answer: There are $2^{19} = 524,288$ subsets. (You need not calculate $2^{19}$.)

   (c) How many subsets of $\mathbb{W}$ have exactly 2 elements?
   Answer: There are "19 choose 2" subsets with 2 elements. The of value of $\binom{19}{2}$ is

$$\binom{19}{2} = \frac{19!}{2!17!} = \frac{19 \cdot 18}{2} = 171$$

Total Points: 45

*When you say the letter, its sound is also a word.