Name:

CSE 1400 Applied Discrete Mathematics

Fall 2012 Quiz Three

Score

1. (15 pts) Consider the sequence of partial sums of the first n natural numbers

$$s_n = \sum_{k=0}^{n-1} k = 0 + 1 + 2 + 3 + \cdots + (n-1) \quad \text{with initial condition } s_0 = 0$$

(a) What are the first 7 terms in the sequence?
Answer:

$$\langle 0,\ 0,\ 1,\ 3,\ 6,\ 10,\ 15 \rangle$$

(b) What recurrence equation does the sequence satisfy?

Answer:

$$s_n = s_{n-1} + (n-1)$$

(c) What function $f(n)$ computes the value of s_n?

Answer:

$$f(n) = \frac{n(n-1)}{2}$$

Score

2. (10 pts) Use mathematical induction to prove

$$\sum_{k=0}^{n-1} \binom{k}{m} = \binom{n}{m+1}$$

Answer:

(a) Establish a basis: For $n = 0$ the sum is empty and equal to zero, and the binomial coefficient $\binom{0}{m+1}$ is zero as well. If you don't like empty proofs: For $n = 1$ the sum is $\binom{0}{m}$ and equal to zero, unless $m = 0$, in which case $\binom{0}{0} = 1$. The binomial coefficient on the right is $\binom{1}{m+1}$ which is equal to zero, unless $m = 0$, in which case $\binom{1}{1} = 1$.

(b) Make a hypothesis: For some $n \geq 0$

$$\sum_{k=0}^{n-1} \binom{k}{m} = \binom{n}{m+1}$$

(c) <u>Use the hypothesis</u>: Consider the equalities

$$\sum_{k=0}^{n} \binom{k}{m} = \left(\sum_{k=0}^{n-1} \binom{k}{m} \right) + \binom{n}{m} \quad \text{definition of summations}$$

$$= \binom{n}{m+1} + \binom{n}{m} \quad \text{by the hypothesis}$$

$$= \binom{n+1}{m+1} \quad \text{Pascal's identity}$$

Total Points: 25