1. (30 pts) Let $a$ and $b$ be integers and let $n$ be a positive integer. Consider the relation “$a$ is congruent to $b$ mod $n$”, written $a \equiv b \pmod{n}$, and meaning $(\exists c \in \mathbb{Z})(a - b = nc)$.

(a) Is “congruence mod $n$” reflexive? Explain your answer.
Answer: Yes, congruence mod $n$ is reflexive: $a - a = 0$ is a multiple of $n$.

(b) Is “congruence mod $n$” symmetric? Explain your answer.
Answer: Yes, congruence mod $n$ is symmetric: If $a - b$ is a multiple of $n$, then $b - a$ is a multiple of $n$.

(c) Is “congruence mod $n$” antisymmetric? Explain your answer.
Answer: No, congruence mod $n$ is not antisymmetric: If $a - b$ is a multiple of $n$ and $b - a$ is a multiple of $n$, it does not follow that $a = b$.

(d) Is “congruence mod $n$” transitive? Explain your answer.
Answer: Yes, congruence mod $n$ is transitive: If $a - b$ is a multiple of $n$ and $b - c$ is a multiple of $n$, then $a - c = (a - b) + (b - a)$ is a multiple of $n$.

(e) Is “congruence mod $n$” an equivalence relation? Explain your answer.
Answer: Yes, congruence mod $n$ is reflexive, symmetric, and transitive.

(f) Is “congruence mod $n$” an partial order? Explain your answer.
Answer: No, congruence mod $n$ is not antisymmetric.

2. (10 pts) How many relations can be defined on a set with cardinality $m$?

Answer: There are $2^{m^2}$ relations.