Name:

CSE 1400 Applied Discrete Mathematics

Fall 2012 Quiz Six

Score

1. (5 pts) Let $(1\ 001\ 1001)_{fp}$ be a normalized floating point number with a sign bit, a 3 bit exponent written as a biased number with bias $b = 4$, and a 4 bit fraction. What is the decimal value of this floating point number?

   Answer: The number is negative since the sign bit is 1. The value of the exponent is $(001)_{b=4} = 1 = 4 = -3$. The normalized fraction is 1.1001 which is equal to

$$1 + \frac{1}{2} + \frac{1}{16} = \frac{25}{16}$$

   Therefore,

$$(1\ 001\ 1001)_{fp} = -\frac{25}{16} \times 2^{-3} = -\frac{25}{128}$$

Score

2. (5 pts) Use the fact that $7 \cdot 90 - 17 \cdot 37 = 1$ to compute $37^{-1} \bmod 90$, the multiplicative inverse of 37 mod 90.

   Answer: The inverse is $-17 \pmod{90}$ which is equal to $-17 + 90 = 73 \pmod{90}$.

Score

3. (5 pts) Use the Euclidean algorithm to compute gcd(37, 90), the greatest common divisor of 37 and 90.

   Answer:

$$\begin{aligned} 90 &= 37 \cdot 2 + 16 \\ 37 &= 16 \cdot 2 + 5 \\ 16 &= 5 \cdot 3 + 1 \\ 5 &= 5 \cdot 1 + 0 \end{aligned}$$

   Therefore, gcd(37, 90) = 1.

Score

4. (5 pts) To compute $7 \cdot 90 - 17 \cdot 37 = 1$, extend the Euclidean algorithm by filling in the "magic table."

37	90
1	0
0	1

   Answer:

37	90	2	2	3	5
1	0	1	2	7	37
0	1	2	5	17	90
	+	−	+	−	+

   Therefore, $7 \cdot 90 - 17 \cdot 37 = 1$.

Score

5. (5 pts) Solve the linear congruence equation

$$37x = 3 \pmod{90}$$

Answer: Since the multiplicative inverse of 37 mod 90 is 73, we have

$$\begin{aligned} x &= 73 \cdot 3 \\ &= 219 \\ &= 219 - 180 \pmod{90} \\ &= 39 \pmod{90} \end{aligned}$$

Note that $37 \cdot 39 = 1443 = 3 \pmod{90}$.

Total Points: 25

Monday, December 3, 2012