1. (5 pts) Let $(10011001)_{fp}$ be a normalized floating point number with a sign bit, a 3 bit exponent written as a biased number with bias $b = 4$, and a 4 bit fraction. What is the decimal value of this floating point number?

**Answer:** The number is negative since the sign bit is 1. The value of the exponent is $(001)_{b=4} = 1 = 4 = -3$. The normalized fraction is $1.1001$ which is equal to

$$1 + \frac{1}{2} + \frac{1}{16} = \frac{25}{16}$$

Therefore,

$$(10011001)_{fp} = \frac{-25}{16} \times 2^{-3} = -\frac{25}{128}$$

2. (5 pts) Use the fact that $7 \cdot 90 - 17 \cdot 37 = 1$ to compute $37^{-1}$ mod 90, the multiplicative inverse of 37 mod 90.

**Answer:** The inverse is $-17$ (mod 90) which is equal to $-17 + 90 = 73$ (mod 90).

3. (5 pts) Use the Euclidean algorithm to compute $\gcd(37, 90)$, the greatest common divisor of 37 and 90.

**Answer:**

$$90 = 37 \cdot 2 + 16$$
$$37 = 16 \cdot 2 + 5$$
$$16 = 5 \cdot 3 + 1$$
$$5 = 5 \cdot 1 + 0$$

Therefore, $\gcd(37, 90) = 1$.

4. (5 pts) To compute $7 \cdot 90 - 17 \cdot 37 = 1$, extend the Euclidean algorithm by filling in the “magic table.”

<table>
<thead>
<tr>
<th>37</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Answer:**

<table>
<thead>
<tr>
<th>37</th>
<th>90</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>37</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>17</td>
<td>90</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Therefore, $7 \cdot 90 - 17 \cdot 37 = 1$. 

1
5. (5 pts) Solve the linear congruence equation

\[ 37x = 3 \pmod{90} \]

Answer: Since the multiplicative inverse of 37 mod 90 is 73, we have

\[
x = 73 \cdot 3 \\
= 219 \\
= 219 - 180 \pmod{90} \\
= 39 \pmod{90}
\]

Note that \(37 \cdot 39 = 1443 = 3 \pmod{90}\).