1. (5 pts) Let \((0 \ 110 \ 1101)_{fp}\) be a normalized floating point number with a sign bit, a 3 bit exponent written as a biased number with bias \(b = 4\), and a 4 bit fraction. What is the decimal value of this floating point number?

Answer: The number is positive since the sign bit is 0. The value of the exponent is \((110)_{b=4} = 164 = 2\). The normalized fraction is 1.1101 which is equal to

\[
1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = \frac{29}{16}
\]

Therefore,

\[
(1 \ 110 \ 1101)_{fp} = \frac{29}{16} \times 2^2 = \frac{29}{4}
\]

2. (5 pts) Use the fact that \(7 \cdot 71 - 16 \cdot 31 = 1\) to compute \(31^{-1} \mod 71\), the multiplicative inverse of 31 mod 71.

Answer: The inverse is \(-16 \mod 71\) which is equal to \(-16 + 71 = 55 \mod 71\).

3. (5 pts) Use the Euclidean algorithm to compute \(\gcd(31, 71)\), the greatest common divisor of 31 and 71.

Answer:

\[
\begin{align*}
71 &= 31 \cdot 2 + 9 \\
31 &= 9 \cdot 3 + 4 \\
9 &= 4 \cdot 2 + 1 \\
4 &= 1 \cdot 4 + 0
\end{align*}
\]

Therefore, \(\gcd(31, 71) = 1\).

4. (5 pts) To compute \(7 \cdot 71 - 16 \cdot 31 = 1\), extend the Euclidean algorithm by filling in the “magic table.”

\[
\begin{array}{c|c}
31 & 71 \\
\hline
1 & 0 \\
0 & 1
\end{array}
\]

Answer:

\[
\begin{array}{cccccc}
31 & 71 & 2 & 3 & 2 & 4 \\
\hline
1 & 0 & 1 & 3 & 7 & 31 \\
0 & 1 & 2 & 7 & 16 & 71 \\
+ & - & + & - & +
\end{array}
\]

Therefore, \(7 \cdot 71 - 16 \cdot 31 = 1\).
5. (5 pts) Solve the linear congruence equation

\[ 31x = 4 \pmod{71} \]

Answer: Since the multiplicative inverse of 31 mod 71 is 55, we have

\[
x = 55 \cdot 4 \\
= 220 \\
= 220 - 213 \pmod{71} \\
= 7 \pmod{71}
\]

Note that \(31 \cdot 7 = 217 = 4 \pmod{71}\).