Name:

CSE 1400 Applied Discrete Mathematics

Fall 2013 Final Exam

1 Boolean Logic

Score

1. (5 pts) Construct a truth table for the Boolean expression

$$\text{C} = (\neg \text{P} \to \text{Q}) \wedge (\neg \text{P} \to \neg \text{Q})$$

Score

2. (5 pts) The complex expression $\text{C} = (\neg \text{P} \to \text{Q}) \wedge (\neg \text{P} \to \neg \text{Q})$ be reduced to what simple expression?

2 Predicate Logic

Score

1. (10 pts) Let $\text{D}(a, b)$ stand for the statement "a (evenly) divides b," where a and b are natural numbers.

   (a) Give example values for a and b to show you understand what $\text{D}(a, b)$ means.

   (b) `True` or `False`? Explain your answer. $(\forall a \in \mathbb{N})(\forall b \in \mathbb{N})(\text{D}(a, b))$

   (c) `True` or `False`? Explain your answer. $(\forall b \in \mathbb{N})(\text{D}(1, b))$

   (d) `True` or `False`? Explain your answer. $(\forall a \in \mathbb{N})(\exists b \in \mathbb{N})(\text{D}(a, b))$

   (e) `True` or `False`? Explain your answer. $(\exists b \in \mathbb{N})(\forall a \in \mathbb{N})(\text{D}(a, b))$

3 Sets

Score

1. (10 pts) Verify the equation $\neg(\mathbb{X} \cap \mathbb{Y}) = \neg\mathbb{X} \cup \neg\mathbb{Y}$ for the sets

$$\begin{aligned}\mathbb{U} &= \{0, 1, 2, 3, 4, 5, 6, 7\}\\ \mathbb{X} &= \{2, 3, 6, 7\}\\ \mathbb{Y} &= \{1, 3, 5, 7\}\end{aligned}$$

4 Counting

Score

1. (10 pts) Tell how to count the following things.

   (a) The number of ways `True` or `False` can be assigned to n Boolean variables.

   (b) The number of different Boolean functions on n Boolean variables.

   (c) The number of different relations on set $\mathbb{X}$ where $|\mathbb{X}| = n$.

   (d) The number of different functions on from $\mathbb{X}$ to $\mathbb{Y}$, where $|\mathbb{X}| = n$ and $|\mathbb{Y}| = m$.

   (e) The number of k-element subsets where the elements are chosen from and n-element set.

5 Mathematical Induction

Score

1. (10 pts) Show that $T(n) = n \lg n$ solves the recurrence equation

$$T_n = 2T_{n/2} + n, \qquad T_1 = 0$$

For $n = 1, 2, 4, 8, 16, \ldots$.

6 Relations

Score

1. (10 pts) Consider the sets $\mathbb{Z}$ and $\mathbb{N}$ of integers and natural numbers. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. If n divides $a - b$, say "a is congruent to b modulo n," write $a \equiv b \pmod{n}$

   Show that congruence modulo n is an equivalence relation.

7 Number Systems

Score

1. (5 pts) Write 53 in binary notation (53 is an unsigned natural number)

Score

2. (5 pts) Write -53 as a two's complement number.

Score

3. (5 pts) Write the normalized floating point number $(1\ 110\ 0101)_{fp}$ in decimal notation.

8 Functions

Score

1. (5 pts) From problem 1 in Number Systems you know $n = 53$ can be written in a certain number of bits. What is this number? Let $b(n) - m$ be the number of bits needed to write a natural number n. What is the formula for $b(n)$?

Score

2. (5 pts) Use Horner's rule to evaluate the polynomial $p(x) = 4x^3 - 5x + 7$ at $x = 2$.

9 Number Theory

Score

1. (5 pts) Use the Euclidean algorithm to compute gcd(42, 101).

Score

2. (5 pts) Use your work in problem 1 (and the magic table) to compute $42^{-1} \bmod 101$.

Score

3. (5 pts) Solve the linear recurrence equation $42x = 3 \bmod 101$.