1. What is the value of \( \lg(\sqrt{8}) \)?
   Answer: \( \lg(\sqrt{8}) = \lg(2^{3/5}) = 3/5 \)

2. Show that \( T(n) = n \lg n \) solves the recurrence equation
   \[
   T_n = 2T_{n/2} + n, \quad T_1 = 0
   \]
   For \( n = 1, 2, 4, 8, 16, \ldots \)
   Answer: First, \( T(1) = 1 \lg 1 = 0 = T_1 \). Next, if
   \[
   T_{n/2} = \frac{n}{2} \lg \left( \frac{n}{2} \right)
   \]
   for some \( n = 2^p \geq 1 \), then
   \[
   2T_{n/2} + n = n \lg \left( \frac{n}{2} \right) + n \\
   = n(\lg n - \lg 2) + n \\
   = n(\lg n - 1) + n \\
   = n \lg n \\
   = T_n
   \]

3. Show that
   \[
   \lceil x \rceil - \lfloor x \rfloor = 0, \quad \text{if } x \in \mathbb{Z}
   \]
   and
   \[
   \lceil x \rceil - \lfloor x \rfloor = 1, \quad \text{if } x \notin \mathbb{Z}
   \]
   Answer: If \( x \) is an integer then \( |x| = x \) and \( \lceil x \rceil = x \), so their difference is 0. If \( x \) is not an integer then \( n < x < n + 1 \) for some integer \( n \) and \( |x| = n \) and \( \lceil x \rceil = n + 1 \), so their difference is 1.