Name:

CSE 1400 — Applied Discrete Mathematics

Fall 2013 — Functions

1. What is the value of $\lg(\sqrt[5]{8})$?
   Answer: $\lg(\sqrt[5]{8}) = \lg(2^{3/5}) = 3/5$

2. Show that $T(n) = n \lg n$ solves the recurrence equation

$$T_n = 2T_{n/2} + n, \qquad T_1 = 0$$

   For $n = 1, 2, 4, 8, 16, \ldots$.
   Answer: First, $T(1) = 1 \lg 1 = 0 = T_1$. Next, if

$$T_{\frac{n}{2}} = \frac{n}{2} \lg\left(\frac{n}{2}\right)$$

   for some $n = 2^p \geq 1$, then

$$\begin{aligned} 2T_{\frac{n}{2}} + n &= n \lg\left(\frac{n}{2}\right) + n \\ &= n(\lg n - \lg 2) + n \\ &= n(\lg n - 1) + n \\ &= n \lg n \\ &= T_n \end{aligned}$$

3. Show that

$$\lceil x \rceil - \lfloor x \rfloor = 0, \qquad \text{if } x \in \mathbb{Z}$$

   and

$$\lceil x \rceil - \lfloor x \rfloor = 1, \qquad \text{if } x \notin \mathbb{Z}$$

   Answer: If x is an integer then $\lfloor x \rfloor = x$ and $\lceil x \rceil = x$, so their difference is 0. If x is not an integer then $n < x < n + 1$ for some integer n and $\lfloor x \rfloor = n$ and $\lceil x \rceil = n + 1$, so their difference is 1.

Total Points: 0 — Wednesday, October 30, 2013