1. Use mathematical induction to show that \( m(n) = 2^n - 1 \) solves the recurrence equation \( m_n = 2m_{n-1} + 1 \) with initial conditions \( m_0 = 0 \) by
   
   (a) Establishing a basis for induction: \( m(0) = m_0 \).
   
   (b) Making a hypothesis that \( m(n-1) = m_{n-1} = 2^{n-1} - 1 \) for some \( n \geq 1 \).
   
   (c) Proving under these assumptions that \( m(n) = m_n = 2^n - 1 \).

2. Use mathematical induction to show that the geometric sum
   \[
   \sum_{k=0}^{n-1} r^k = \frac{r^n - 1}{r - 1}
   \]
   provided \( r \neq 1 \).

3. Use mathematical induction to prove the sum of products consecutive pairs of natural numbers is the product of three consecutive number divided by 3, that is,
   \[
   \sum_{k=0}^{n-1} k(k-1) = \frac{n(n-1)(n-2)}{3}
   \]
   Notice that this identity states
   \[
   \sum_{k=0}^{n-1} \binom{k}{2} = \binom{n}{3}
   \]