1. (10 pts) Let $p$, $q$, and $r$ be Boolean variables. If $p = \text{True}$, $q = \text{False}$, and $r = \text{False}$, what are the values of the following expressions?

   (a) $\neg (q \lor r)$

   (b) $\neg (p \land q) \lor r$

2. (5 pts) In how many ways can truth values be assigned to $n$ Boolean variables?

3. (5 pts) How many different $n$-variable Boolean functions are there?

4. (10 pts) Construct a truth table for the Boolean expression (function)

   $((\neg p \rightarrow q) \land (\neg p \rightarrow \neg q)) \rightarrow p$
5. (5 pts) What Boolean expression does the shaded region represent?

\[ \text{Q} \cup \text{P} \]

6. (15 pts) Let

\[ \mathbb{D} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \] be the universe of digits
\[ \mathbb{E} = \{0, 2, 4, 6, 8\} \] be the even digits
\[ \mathbb{O} = \{1, 3, 5, 7, 9\} \] be the odd digits
\[ \mathbb{P} = \{2, 3, 5, 7\} \] be the prime digits.

(a) Is \( \mathbb{O} \cup (\mathbb{E} \cap \mathbb{P}) = (\mathbb{O} \cup \mathbb{E}) \cap \mathbb{P} \) True or False? Explain your answer.

(b) Verify De Morgan’s law: \( \neg(\mathbb{O} \cap \mathbb{P}) = \neg\mathbb{O} \cup \neg\mathbb{P} \)

(c) What is (describe) \( 2^{\mathbb{D}} \)?

7. (10 pts) I once gave a 20 question True/False exam.

(a) In how many ways can you answer the questions (pretend you answer each question True or False)?

(b) If you decide to answer one-half of the questions True and one-half False, in how many ways can you answer the questions?
8. (10 pts) What is Pascal’s identity and what are its boundary conditions?

9. (5 pts) Let \( X = \{a, b, c, d, e\} \). What is the notation for the Stirling number of the second kind, which counts partitions of \( X \) into 3 subsets? (Extra credit (5 pts)) What is the Stirling’s second recurrence equation for counting partitions?

10. (10 pts) Consider the sequence \( \vec{M} = \langle 0, 1, 3, 7, 15, 31, 63, \ldots \rangle \).
   (a) What is the function \( m(n) \) that computes the terms \( m_n \) for \( n \in \mathbb{N} \)?
   
   (b) What is the recurrence equation that terms in the sequence satisfy?
11. (5 pts) Write the first 6 terms in the sequence defined by the recurrence equation and initial condition.

\[ f_n = f_{n-1} - f_{n-2}, \quad f_0 = 0, \quad f_1 = 1 \]

12. (5 pts) Consider the summation

\[ S_n = \sum_{k=0}^{n-1} (4k + 3) \]

Show that the function \( S(n) = n(2n + 1) \) satisfies the recurrence equation \( S_{n+1} = S_n + (4n + 3) \).

13. (5 pts) Use mathematical induction to prove the following summation formula.

\[ \sum_{k=0}^{n-1} k(k - 1) = \frac{n(n - 1)(n - 2)}{3} \]