Name:

CSE 1400 — Applied Discrete Mathematics

Fall 2013 — Number Systems

1. `True` or `False`: The largest positive integer that can be represented with p bits is

$$1 + 2 + 2^2 + \cdot + 2^{p-1} = 2^p - 1$$

Answer: This is `True`. The largest positive 2-bit integer is $(11)_2 = 3 = 2^2 - 1$; The largest positive 3-bit integer is $(111)_2 = 7 = 2^3 - 1$; and in general, the largest p-bit integer is a string of 1's of length p which represents

$$1 + 2 + 2^2 + \cdot + 2^{p-1} = 2^p - 1$$

2. `True` or `False`: If n is an positive integer, then $2^p \leq n < 2^{p+1}$ for some power $p = 0, 1, 2, \ldots$ of 2.

Answer: This is `True`. If n is a positive integer then $1 = 2^0 \leq n < 2^1 = 2$ or $2 = 2^1 \leq n < 2^2 = 4$ or $4 = 2^2 \leq n < 2^3 = 8$, and so on.

3. `True` or `False`: If $2^p \leq n < 2^{p+1}$ then n can be represented using $p + 1$ bits.

Answer: This is `True`. If $n < 2^{p+1}$, then $n \leq 2^{p+1} - 1$ and by problem (1), n can be written using $p+1$ bits.

4. `True` or `False`: A positive integer n can be represented using

$$\lfloor \lg n \rfloor + 1 \quad \text{bits.}$$

Answer: This is `True`. Solve $2^p \leq n < 2^{p+1}$ for $p + 1$ to obtain $p \leq \lg n < p + 1$ so that

$$p = \lfloor \lg n \rfloor \qquad \text{and} \qquad \lfloor \lg n \rfloor + 1 = p + 1$$

5. Use Horner's rule to convert the binary number $(0100\ 1001)_2$ to decimal notation.

Answer:

Horner's Rule							
0	1	0	0	1	0	0	1
		2	4	8	18	36	72
	1	2	4	9	18	36	$\boxed{73}$

Therefore, $(0100\ 1001)_2 = (73)_{10}$

Total Points: 0 — Monday, November 4, 2013