1. **True or False**: The largest positive integer that can be represented with \( p \) bits is

\[
1 + 2 + 2^2 + \cdots + 2^{p-1} = 2^p - 1
\]

**Answer**: This is True. The largest positive 2-bit integer is \((11)_2 = 3 = 2^2 - 1\); The largest positive 3-bit integer is \((111)_2 = 7 = 2^3 - 1\); and in general, the largest \( p \)-bit integer is a string of 1’s of length \( p \) which represents

\[
1 + 2 + 2^2 + \cdots + 2^{p-1} = 2^p - 1
\]

2. **True or False**: If \( n \) is an positive integer, then \( 2^p \leq n < 2^{p+1} \) for some power \( p = 0, 1, 2, \ldots \) of 2.

**Answer**: This is True. If \( n \) is a positive integer then \( 1 = 2^0 \leq n < 2^1 = 2 \) or \( 2 = 2^1 \leq n < 2^2 = 4 \) or \( 4 = 2^2 \leq n < 2^3 = 8 \), and so on.

3. **True or False**: If \( 2^p \leq n < 2^{p+1} \) then \( n \) can be represented using \( p + 1 \) bits.

**Answer**: This is True. If \( n < 2^{p+1} \), then \( n \leq 2^{p+1} - 1 \) and by problem (1), \( n \) can be written using \( p + 1 \) bits.

4. **True or False**: A positive integer \( n \) can be represented using

\[
\lfloor \log n \rfloor + 1 \text{ bits.}
\]

**Answer**: This is True. Solve \( 2^p \leq n < 2^{p+1} \) for \( p + 1 \) to obtain \( p \leq \log n < p + 1 \) so that

\[
p = \lfloor \log n \rfloor \quad \text{and} \quad \lfloor \log n \rfloor + 1 = p + 1
\]

5. Use Horner’s rule to convert the binary number \((0100 1001)_2\) to decimal notation.

**Answer**:

\[
\begin{array}{c|c|c|c|c|c|c|c}
\text{Horner’s Rule} \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
\hline
2 & 4 & 8 & 18 & 36 & 72 \\
\hline
1 & 2 & 4 & 9 & 18 & 36 & 73 \\
\end{array}
\]

Therefore, \((0100 1001)_2 = (73)_{10}\)