1. (25 pts) Let

\[ D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \text{ be the universe of digits} \]
\[ E = \{0, 2, 4, 6, 8\} \text{ be the even digits} \]
\[ O = \{1, 3, 5, 7, 9\} \text{ be the odd digits} \]
\[ P = \{2, 3, 5, 7\} \text{ be the prime digits}. \]

(a) What is the set \( E \cap O \)?

(b) Is \( O \cup (E \cap P) = (O \cup E) \cap P \) True or False? Explain your answer.

(c) Verify De Morgan’s law: \( \neg(O \cap P) = \neg O \cup \neg P \)

(d) What is \( |D| \)?

(e) What is \( 2^D \)?
2. (20 pts) I once gave a 20 question True/False exam.

(a) In how many ways can you answer the questions (pretend you answer each question True or False)?

(b) If you decide to leave some questions blank, in how many ways can you answer the questions?

(c) If you decide to answer one-half of the questions True and one-half False, in how many ways can you answer the questions?

(d) In how many ways could the questions be partitioned into 3 subsets: Those answered True, those answered False, and those left blank.
3. (5 pts) With respect to sets and subsets, what does the number \( \binom{12}{5} \) represent?

4. (10 pts) What is Pascal’s identity and what are its boundary conditions?

5. (10 pts) What is the sum of values in row \( n \) in Pascal’s triangle? Why is it this expression?

6. (10 pts) Let \( X = \{a, b, c, d, e\} \).
   
   (a) List a partition of \( X \) into 3 subsets.

   (b) What is the notation for the number of partitions of \( X \) into 3 subsets?
7. (5 pts) Here is a Venn diagram. What set expressions does the shaded region represent?

8. (5 pts) What Boolean expression does the shaded region represent?

9. (5 pts) How many different 3-variable Boolean functions are there?

10. (5 pts) In how many ways could you shade the 3-circle Venn diagram?

Total Points: 100