Name: **Class ID:**

CSE 1400 Applied Discrete Mathematics

Fall 2013 Quiz 3

Score

1. (20 pts) Write the first 6 terms in the sequence defined by the recurrence equation and initial condition.

$$f_n = f_{n-1} - f_{n-2}, \qquad f_0 = 0,\ f_1 = 1$$

Answer: $f_0 = 0,\ f_1 = 1,\ f_2 = f_1 - f_0 = 1,\ f_3 = f_2 - f_1 = 0,\ f_4 = f_3 - f_2 = -1,\ f_5 = f_4 - f_3 = -1.$

Score

2. (20 pts) Show that the function $f(n) = n(n-1)/2$ solves the recurrence equation $f_n = f_{n-1} + (n-1)$ and initial condition $f_0 = 0$.

Answer: Note that $f(0) = 0(0-1)/2 = 0 = f_0$ so that the initial condition is satisfied. If $f(n) = n(n-1)/2$, then $f(n-1) = (n-1)(n-2)/2$ and

$$\begin{aligned} f(n-1) + (n-1) &= (n-1)\left[\frac{n-2}{2} + 1\right] \\ &= (n-1)\left[\frac{n-2}{2} + \frac{2}{2}\right] \\ &= \frac{n(n-1)}{2} \\ &= f(n) \end{aligned}$$

(Problems continue on the back of this page)

Score

3. (20 pts) Consider writing a string of bits so that there are no consecutive 1's, for example 100101 is such a string and its length is 6.

   (a) Write all bit strings of length one that do not have consecutive 1's.

   Answer: 0 and 1 do not have consecutive 1.

   (b) Show how to use the two-bit string 01 together with your answers from problem (3a) to make 3 bit strings, without consecutive 1's.

   Answer: $0 \to 001$ and $1 \to 101$.

   (c) Write all bit strings of length two that do not have consecutive 1's.

   Answer: 01, 00, and 10 do not have consecutive 1.

   (d) Find a single bit that you can append to your answers in problem (3c) to make 3 bit strings, without consecutive 1's.

   Answer: $01 \to 010$, $00 \to 000$, $10 \to 100$.

   (e) Let b_n be the number of bits strings of length n without consecutive 1. Use the results of problems (3d) and (3b) to derive a recurrence relation for b_n in terms of b_{n-1} and b_{n-2}.

   Answer: Notice strings of length $n-2$ can be extend to strings of length n by appending 01: If the original string does not have consecutive 1, then neither will the new string. Similarly, strings of length $n-1$ can be extend to strings of length n by appending 0: If the original string does not have consecutive 1, then neither will the new string. Also, these new strings will be different because the first ones end in 1 and the second ones end in 0.

Score

4. (20 pts) Consider the sequence $\vec{M} = \langle 0, 1, 3, 7, 15, 31, 63, \ldots \rangle$.

   (a) What is the function $m(n)$ that computes the terms m_n for $n \in \mathbb{N}$?

   Answer: The terms can be computed by the function $m(n) = 2^n - 1$.

   (b) What is the recurrence equation that terms in the sequence satisfy?

   Answer: The recurrence equation is $m_n = 2m_{n-1} + 1$.

Score

5. (20 pts) Consider the summation

$$S_n = \sum_{k=0}^{n-1}(4k+3)$$

(a) Show that $S_{n+1} = S_n + (4n+3)$

Answer: If $S_n = \sum_{k=0}^{n-1}(4k+3)$, then $S_{n+1} = \sum_{k=0}^{n}(4k+3) = \sum_{k=0}^{n-1}(4k+3) + (4n+3) = S_n + (4n+3)$.

(b) Show that the function $S(n) = n(2n+1)$ satisfies the recurrence equation $S_{n+1} = S_n + (4n+3)$.

Answer: If $S(n) = n(2n+1)$, then $S(n+1) = (n+1)(2n+3)$ and

$$\begin{aligned} S(n) + (4n+3) &= (2n^2+2n) + (4n+3) \\ &= 2n^2 + 6n + 3 \\ &= (n+1)(2n+3) \end{aligned}$$