Name:

CSE 1400 Applied Discrete Mathematics
Fall 2013 Quiz 5

Score

1. (30 pts) Consider the sets

$$\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \ldots\} \qquad \mathbb{N} = \{0, 1, 2, 3, 4, 5, \ldots\}$$

of integers and natural numbers. Let a, $b \in \mathbb{Z}$ and let $n \in \mathbb{N}$. If n divides $a - b$, write

$$a \equiv b \pmod{n}$$

and say "a is congruent to b modulo n."

Show that congruence modulo n is an equivalence relation.

Answer: Congruence is

- Reflexive: $(\forall a \in \mathbb{Z})(a \equiv a \pmod{n})$ because n divides $a - a = 0$.
- Symmetric: $(\forall a, b \in \mathbb{Z})((a \equiv b \pmod{n}) \to (b \equiv a \pmod{n}))$ because if n divides $a - b$, then n divides $b - a$. That is, if $a - b = n \cdot q$ for some quotient $q \in \mathbb{Z}$, then $b - a = n(-q)$.
- Transitive: $(\forall a, b, c \in \mathbb{Z})((a \equiv b \pmod{n}) \wedge (b \equiv c \pmod{n}) \to (a \equiv c \pmod{n}))$ because if n divides $a - b$ and n divides $b - c$, then n divides $a - c$. That is, if $a - b = n \cdot q$ for some quotient $q \in \mathbb{Z}$ and $b - c = n \cdot q'$, then $a - c = (a - b) + (b - c) = n(q + q')$, and n divides $a - c$.

Score

2. (20 pts) Use Horner's rule to evaluate the polynomial $p(x) = 4x^3 - 5x + 7$ at $x = 2$.

Answer:

Horner's Rule			
4	0	-5	7
	8	16	22
4	8	11	$\boxed{29}$

Score

3. (20 pts) Show that $T(n) = n \lg n$ solves the recurrence equation

$$T_n = 2T_{n/2} + n, \qquad T_1 = 0$$

For $n = 1, 2, 4, 8, 16, \ldots$.

Answer: First, $T(1) = 1 \lg 1 = 0 = T_1$. Next, if

$$T_{\frac{n}{2}} = \frac{n}{2} \lg \left(\frac{n}{2}\right)$$

for some $n = 2^p \geq 1$, then

$$\begin{aligned} 2T_{\frac{n}{2}} + n &= n \lg \left(\frac{n}{2}\right) + n \\ &= n(\lg n - \lg 2) + n \\ &= n(\lg n - 1) + n \\ &= n \lg n \\ &= T_n \end{aligned}$$

Score

4. (20 pts) Consider the set $\mathbb{D} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   (a) Write the permutation $\langle 9, 7, 5, 3, 1, 0, 2, 4, 6, 8 \rangle$ of elements in $\mathbb{D}$ using cyclic notation.
   Answer: Write the elements in their natural order above their permuted order:

0	1	2	3	4	5	6	7	8	9
9	7	5	3	1	0	2	4	6	8

   Then read off the movement of elements

$$[0, 5, 2, 6, 8, 9][1, 4, 7][3]$$

   (b) How many cycles are there in the permutation?
   Answer: There are 3 cycles.

Score

5. (10 pts) How many bits are needed to write 573 in binary?

   Answer: The number of bits needed to represent n is given by the formula

$$\lfloor \lg n \rfloor + 1$$

   In this case, since $512 = 2^9 < 573 < 2^{10} = 1024$, the floor of the log (base 2) of 573 is 9. Therefore 10 bits are required to write 573 in binary.

Total Points: 100 Wednesday, November 6, 2013