Name: **Class ID:**

CSE 1400 Applied Discrete Mathematics
Fall 2013 Recursion

1. Show that the given sum solves the recurrence equation and matches the initial sum.

   (a) Sum: $T_n = 0 + 1 + 2 + 3 + \cdots + (n-1)$,
   Recurrence equation: $t_n = t_{n-1} + (n-1)$,
   Initial sum: $t_0 = 0$.
   Answer: If
   $$T_n = 0 + 1 + 2 + 3 + \cdots + (n-1) = \sum_{k=0}^{n-1} k$$
   then
   $$T_{n-1} = 0 + 1 + 2 + 3 + \cdots + (n-2) = \sum_{k=0}^{n-2} k$$
   and
   $$\begin{aligned} T_{n-1} + (n-1) &= [0 + 1 + 2 + 3 + \cdots + (n-2)] + (n-1) \\ &= T_n \end{aligned}$$
   Also, the initial condition is satisfied $T_0 = 0$, because this sum is empty.

   (b) Sum: $M_n = 1 + 2 + 2^2 + \cdots + 2^{n-1}$,
   Recurrence equation: $m_n = 2m_{n-1} + 1$,
   Initial sum: $m_0 = 0$.
   Answer: If
   $$M_n = 1 + 2 + 2^2 + \cdots + 2^{n-1} = \sum_{k=0}^{n-1} 2^k$$
   then
   $$M_{n-1} = 1 + 2 + 2^2 + \cdots + 2^{n-2} = \sum_{k=0}^{n-2} 2^k$$
   and
   $$\begin{aligned} 2M_{n-1} + 1 &= [2 + 4 + 2^3 + \cdots + 2^{n-1}] + 1 \\ &= M_n \end{aligned}$$
   Also, the initial condition is satisfied $M_0 = 0$, because this sum is empty.

2. Let $\vec{X} = \langle x_0, x_1, x_2, \ldots \rangle$ be a sequence of numbers. Consider the sum
   $$S_n = \sum_{k=0}^{n-1} x_k$$

   (a) What is the value of S_0?
   Answer: S_0 is the empty sum, and its value is 0.

(b) Write a recurrence equation for S_{n+1} using S_n and a term the sequence $\vec{X}$.
Answer:

$$S_{n+1} = S_n + x_n$$