1. Show that the given sum solves the recurrence equation and matches the initial sum.

(a) Sum: \( T_n = 0 + 1 + 2 + 3 + \cdots + (n - 1) \),
Recurrence equation: \( t_n = t_{n-1} + (n - 1) \),
Initial sum: \( t_0 = 0 \).
Answer: If \( T_n = 0 + 1 + 2 + 3 + \cdots + (n - 1) = \sum_{k=0}^{n-1} k \) then
\[ T_{n-1} = 0 + 1 + 2 + 3 + \cdots + (n - 2) = \sum_{k=0}^{n-2} k \]
and
\[ T_{n-1} + (n - 1) = [0 + 1 + 2 + 3 + \cdots + (n - 2)] + (n - 1) = T_n \]

Also, the initial condition is satisfied \( T_0 = 0 \), because this sum is empty.

(b) Sum: \( M_n = 1 + 2 + 2^2 + \cdots + 2^{n-1} \),
Recurrence equation: \( m_n = 2m_{n-1} + 1 \),
Initial sum: \( m_0 = 0 \).
Answer: If \( M_n = 1 + 2 + 2^2 + \cdots + 2^{n-1} = \sum_{k=0}^{n-1} 2^k \) then
\[ M_{n-1} = 1 + 2 + 2^2 + \cdots + 2^{n-2} = \sum_{k=0}^{n-2} 2^k \]
and
\[ 2M_{n-1} + 1 = [2 + 4 + 2^3 + \cdots + 2^{n-1}] + 1 = M_n \]

Also, the initial condition is satisfied \( M_0 = 0 \), because this sum is empty.

2. Let \( \bar{X} = \langle x_0, x_1, x_2, \ldots \rangle \) be a sequence of numbers. Consider the sum

\[ S_n = \sum_{k=0}^{n-1} x_k \]

(a) What is the value of \( S_0 \)?
Answer: \( S_0 \) is the empty sum, and its value is 0.
(b) Write a recurrence equation for $S_{n+1}$ using $S_n$ and a term the sequence $\vec{X}$.

Answer:

$$S_{n+1} = S_n + x_n$$