Name: **Class ID:**

CSE 1400 Applied Discrete Mathematics
Fall 2013 Induction

1. Consider the set of integers
$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \ldots\}.$$
and the set of natural numbers
$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, \ldots\}.$$
Let $a, b \in \mathbb{Z}$ and let $n \in \mathbb{N}$. If n divides $a - b$, write
$$a \equiv b \pmod{n}$$
and say a is congruent to b modula n.

   - Show that conguence modulo n is an equivalence relation.
     Answer: Congruence is
     - Reflexive: $(\forall a \in \mathbb{Z})(a \equiv a \pmod{n})$ because n divides $a - a = 0$.
     - Symmetric: $(\forall a, b \in \mathbb{Z})((a \equiv b \pmod{n}) \to (b \equiv a \pmod{n}))$ because if n divides $a - b$, then n divides $b - a$. That is, if $a - b = n \cdot q$ for some quotient $q \in \mathbb{Z}$, then $b - a = n(-q)$.
     - Transitive: $(\forall a, b, c \in \mathbb{Z})((a \equiv b \pmod{n}) \wedge (b \equiv c \pmod{n}) \to (a \equiv c \pmod{n}))$ because if n divides $a - b$ and n divides $b - c$, then n divides $a - c$. That is, if $a - b = n \cdot q$ for some quotient $q \in \mathbb{Z}$ and $b - c = n \cdot q'$, then $a - c = (a - b) + (b - c) = n(q + q')$, and n divides $a - c$.
   - What does $a \equiv b \pmod{1}$ mean?
     Answer: If $a \equiv b \pmod{1}$ then $a - b$ is divisible by 1. Since 1 divides every number, $a \equiv b \pmod{1}$ means every integer a is related to every integer b.
   - What does $a \equiv b \pmod{0}$ mean?
     Answer: If $a \equiv b \pmod{0}$ then $a - b$ is divisible by 0. The only number 0 divides is 0, so $a \equiv b \pmod{0}$ means $a = b$.

Total Points: 0 Friday, October 18, 2013