1. Using the notation of predicate logic, write the definition that \( f : X \rightarrow Y \) is a function from \( X \) to \( Y \).
   Answer:
   \[
   (\forall x \in X)(\forall y_0, y_1 \in Y)((f(x) = y_0) \land (f(x) = y_1) \rightarrow (y_0 = y_1))
   \]

2. Using the notation of predicate logic, write the definition that \( f : X \rightarrow Y \) is an onto function from \( X \) to \( Y \).
   Answer:
   \[
   (\forall y \in Y)(\exists x \in X)(f(x) = y)
   \]

3. Using the notation of predicate logic write the definition that \( f : X \rightarrow Y \) is a one-to-one function from \( X \) to \( Y \).
   Answer:
   \[
   (\forall u \in X)(\forall v \in X)((u \neq v) \rightarrow (f(u) \neq f(v)))
   \]

4. Do you know your conic sections? Which of the formulas define functions from \( X \) to \( Y \)?
   (a) The ellipse: \( (\frac{x}{a})^2 + (\frac{y}{b})^2 = 1 \)
   Answer: The elliptical curve is not a function from \( X \) to \( Y \). For instance, when \( x = 0 \), the value of \( y \) can be \( b \) or \( -b \). A function from \( X \) to \( Y \) maps each \( x \) to one and only one \( y \).
   (b) The hyperbola: \( (\frac{x}{a})^2 - (\frac{y}{b})^2 = 1 \)
   Answer: This hyperbolic curve is not a function from \( X \) to \( Y \). For instance, when \( x = \sqrt{2a} \), the value of \( y \) can be \( b \) or \( -b \). A function from \( X \) to \( Y \) maps each \( x \) to one and only one \( y \).
   (c) The parabola: \( x^2 = 4ay \)
   Answer: This parabolic curve is a function from \( X \) to \( Y \). Each \( x \) maps to one and only one \( y \).

Check your answers
Discuss answers with other students, course assistants, and the professor.

Total Points: 0