1. Let $x$ and $y$ be integers ($x, y \in \mathbb{Z}$).

(a) Is the statement: “There is an $x$ such that $x + y = 1$” True or False or undecidable? Written mathematically,

$$(\exists x \in \mathbb{Z})(x + y = 1)$$

(b) If statement (1) is True, what is an $x$ that makes $x + y = 1$?

(c) If statement (1) is False, why can’t there be an $x$ such $x + y = 1$?

(d) If statement (1) is undecidable, describe the information needed to decide it?

2. Is the statement: “For every $y$, there is an $x$ such that $x + y = 1$” True or False or undecidable?

Written mathematically,

$$(\forall y \in \mathbb{Z})(\exists x \in \mathbb{Z})(x + y = 1)$$

Explain your answer.

3. Is statement: “There is an $x$ such that for every $y$, $x + y = 1$” True or False or undecidable?

Written mathematically,

$$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x + y = 1)$$

Explain your answer.

4. Now, lets restrict $x$ and $y$ be natural numbers ($x, y \in \mathbb{N}$).

(a) Is the statement: “There is an $x$ such that $x + y = 1$” True or False or undecidable? Written mathematically,

$$(\exists x \in \mathbb{N})(x + y = 1)$$

(b) Is the statement: “For every $y$, there is an $x$ such that $x + y = 1$” True or False or undecidable? Written mathematically,

$$(\forall y \in \mathbb{N})(\exists x \in \mathbb{N})(x + y = 1)$$

Explain your answer.