Name:

CSE 1400 Applied Discrete Mathematics
Spring 2014 Quiz 1

Score

1. (10 pts) Fill in the truth table for De Morgan's first law: If P and Q are not both `True`, then one of P or Q is `False`. Conversely, if one of P or Q is `False`, then not both of them are `True`.

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

De Morgan's First Law

Input		Computations	
P	Q	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Score

2. (10 pts) Construct a truth table for the Boolean expression.

$$[(P \to Q) \wedge (\neg P \to R)] \to (Q \vee R)$$

Answer:

Input						Output	
P	Q	R	$(P \to Q)$	$\wedge$	$(\neg P \to R)$	$\to$	$(Q \vee R)$
0	0	0	1	0	0	1	0
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	1	1	0
1	0	1	0	0	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

Score

3. (10 pts) Given the truth table below, find a Boolean expression that computes it.

	Input			Output
Row	A	B	C	D
1	0	0	0	0
2	0	0	1	0
3	0	1	0	0
4	0	1	1	1
5	1	0	0	0
6	1	0	1	1
7	1	1	0	1
8	1	1	1	1

Answer: Taking the row where the output D is 1. Form an AND-clauses of the input to make the row `True`. Form the OR of these clauses.

$$\mathrm{D} = (\neg\mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C}) \vee (\mathrm{A} \wedge \neg\mathrm{B} \wedge \mathrm{C}) \vee (\mathrm{A} \wedge \mathrm{B} \wedge \neg\mathrm{C}) \vee (\mathrm{A} \wedge \mathrm{B} \wedge \mathrm{C})$$

Alternatively, take the row where the output D is 0. Form an OR-clauses of the input to make the row `False`. Form the AND of these clauses.

$$\mathrm{D} = (\mathrm{A} \vee \mathrm{B} \vee \mathrm{C}) \wedge (\mathrm{A} \vee \mathrm{B} \vee \neg\mathrm{C}) \wedge (\mathrm{A} \vee \neg\mathrm{B} \vee \mathrm{C}) \wedge (\neg\mathrm{A} \vee \mathrm{B} \vee \mathrm{C})$$

Score

4. (10 pts) The Internet Protocol (IP) is used to route traffic on the Internet. IPv6 uses 128-bits (16-bytes) to name addresses. What is the size of Internet version 6 space?

Answer: Internet version 6 space has $2^{128} = 16^{32}$ addresses. A simple approximation gives

$$\begin{aligned} 2^{128} &= \left(2^{10}\right)^{12.8} \\ &\approx \left(10^{3}\right)^{12.8} \\ &= 10^{38.4} \end{aligned}$$

SI prefixes go up to yotta: 10^{24}. So there are about one hecto-tera-yotta-addresses (10^{38}) in IPv6 space.

Score

5. (10 pts) Using n digits what is the largest natural number that can be written? Using n bits what is the largest natural number that can be written?

   Answer: The largest decimal number that can written using n digits is $10^n - 1$. The largest binary number that can written using n bits is $2^n - 1$.

Score

6. (10 pts) How many bits does it take to write the number 73? How many bits does it take to write the number n?

   Answer: 73 can be written in 7 bits. Note that repeated division (quotients and remainders) is a simple way to compute a number's binary representation. For instance,

Repeated Remaindering Mod 2							
Quotients	73	36	18	9	4	2	1
Remainders	1	0	0	1	0	0	1

   Therefore $73 = (1001001)_2$.

   In general, any number $n \geq 1$ can be bound below and above by powers of 2

$$2^k \leq n < 2^{k+1} \quad \text{for some } k \geq 0.$$

   In this case, it will require $k+1$ bits to write n. Using the inequalities above, and taking the log base 2 of each term, deduce that

$$k \leq \lg n < k + 1$$

   Take the floor of $\lg n$ to get k and add 1 to get $k+1$, the number of bits needed to write n.

$$\lfloor \lg n \rfloor + 1 = \text{number of bits to write } n.$$

Score

7. (30 pts) Let n, $m \in \mathbb{N}$ be natural numbers. Write the following statements using mathematical notation.

   (a) There is an $n \in \mathbb{N}$ such that $m = n/2$.
   Answer:
   $$(\exists n \in \mathbb{N})(m = n/2)$$

   (b) For all $m \in \mathbb{N}$, there is an $n \in \mathbb{N}$ such that $n/2 = m$.
   Answer:
   $$(\forall m \in \mathbb{N})(\exists n \in \mathbb{N})(m = n/2)$$

   (c) There is an $n \in \mathbb{N}$ such that for all $m \in \mathbb{N}$, $m = n/2$.
   Answer:
   $$(\exists n \in \mathbb{N})(\forall m \in \mathbb{N})(m = n/2)$$

   Which of these three statements are `True`, `False`, or `undecidable`.

   Answer:

   (a) Question (7a) is `True`: $n = 2m$ is a natural number if m is and $m = n/2$.
   (b) Question (7b) is `True`: Since there were no restrictions on m in the previous answer, the argument applies to all m.
   (c) Question (7c) is `False`: It says there is some fixed natural number n such that every other natural number is twice n ($m = 2n$).

Score

8. (10 pts) Okay, I have to know, who can solve my favorite quadratic equation?
   $$x^2 - x - 1 = 0$$
   The roots are interesting numbers, for 10 extra points, tell me about the roots.

Total Points: 100 Wednesday, January 22, 2014